New Variable Step Size Affine Projection Algorithms

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***Abstract*—In this paper, we propose new variable step size affine projection algorithms whose step sizes are adjusted according to the square of a time-averaging estimate of the autocorrelation of a priori and a posteriori errors. The proposed algorithms have fast convergence, robustness against near-end signal variations (including double-talk) and do not require any a priori information about the acoustic environment. The simulation results indicate the good performance of the proposed algorithms when compared to similar algorithms.**

***Keywords*- adaptive filters, affine projection algorithm, variable step size, echo cancellation.**

I. INTRODUCTION

The main part of the echo cancellation application can be interpreted as a system identification problem, where an adaptive filter is used to identify an unknown system, i.e., the echo path [1]. The affine projection algorithm (APA) [2] and some of its versions, e.g., [3-6], were found to be very attractive choices for AEC applications, since they offer a superior convergence rate as compared to the normalized least mean square (NLMS) algorithm, especially for speech signals. The performance of the classical APA is governed by the step-size parameter. This parameter has to be chosen based on a compromise between fast convergence rate and good tracking capabilities on the one hand, and low misadjustment on the other hand. In [7-8], non-parametric variable step size (VSS) NLMS algorithms were proposed. A number of variable step-size APAs (VSS-APAs) were developed too (e.g in [9-12] and reference therein). Usually, a double-talk detector (DTD) is used in order to slow down or completely halt the adaptation process during double-talk periods [13-14].

In this paper, we propose new non-parametric VSS-APA (NVSS-APA). In the proposed approach, the step size is modified taking into account the square of the square of the time-averaged estimation of the autocorrelation of *a priori* and *a posteriori* errors. The resulting formula of the step size depends only on signals that are available within the AEC application. We also derive a version based on dichotomous coordinate descent [15-16] (DCD) called NVSS-APA-DCD.

The paper is organized as follows. Section 2 introduces the classical APA, followed by the derivation of the proposed NVSS-APA and NVSS-APA-DCD. The simulation results are presented in Section 3. Finally, Section 4 concludes this work.

II. PROPOSED ALGORITHMS

An unknown system is identified using an adaptive filter and both have finite impulse responses, defined by the real-valued vectors $\mathbf{h} = [h_0\ h_1\ \ldots\ h_{N-1}]^T$ and $\hat{\mathbf{h}}(n) = [\hat{h}_0(n)\ \hat{h}_1(n)\ \ldots\ \hat{h}_{L-1}(n)]^T$, where superscript T denotes transposition and n is the time index; N is the length of the echo path, while L is the length of the adaptive filter. The signal $x(n)$ is the far-end speech which goes through the acoustic impulse response $\mathbf{h}$, resulting the echo signal, $y(n)$. This signal is picked up by the microphone together with the near-end signal $v(n)$, resulting the microphone signal $d(n)$. The near-end signal can contain both the background noise, $w(n)$, and the near-end speech, $u(n)$. The output of the adaptive filter, $\hat{y}(n)$, provides a replica of the echo, which will be subtracted from the microphone signal.

The APA [2] is defined by the following relations:

$$\mathbf{e}(n) = \mathbf{d}(n) - \mathbf{X}^T(n)\hat{\mathbf{h}}(n-1)\,, \tag{1}$$

$$\hat{\mathbf{h}}(n) = \hat{\mathbf{h}}(n-1) + \mathbf{X}(n)\left[\delta\mathbf{I}_p + \mathbf{X}^T(n)\mathbf{X}(n)\right]^{-1}\boldsymbol{\mu}(n)\mathbf{e}(n) \tag{2}$$

where

$$\boldsymbol{\mu}(n) = \mathrm{diag}\left\{\mu_0(n), \mu_1(n), \ldots, \mu_{p-1}(n)\right\} \tag{3}$$

is a $p \times p$ diagonal matrix, δ is the regularization factor, $\mathbf{e}(n)$ is the a priori error vector and $\mathbf{d}(n) = [d(n), d(n-1), \ldots, d(n-p+1)]T$ is the desired signal vector of length p, with p denoting the projection order. The matrix $\mathbf{X}(n) = [\mathbf{x}(n), \mathbf{x}(n-1), \ldots, \mathbf{x}(n-p+1)]$ is the input signal matrix, where $\mathbf{x}(n-l) = [x(n-l), x(n-l-1), \ldots, x(n-l-L+1)]^T$ (with $l = 0,1,\ldots, p-1$) are the input signal vectors. The constant μ denotes the step-size parameter of the algorithm.

Using the adaptive filter coefficients at time n, the a posteriori error vector can be defined as

$$\boldsymbol{\varepsilon}(n) = \mathbf{d}(n) - \mathbf{X}^T(n)\hat{\mathbf{h}}(n)\,. \tag{4}$$

It can be noticed that the vector $\mathbf{e}(n)$ from (1) plays the role of the a priori error vector. Replacing (2) in (4) and taking (1) into account, it results that

$$\boldsymbol{\varepsilon}(n)=\left[\mathbf{I}_p-\boldsymbol{\mu}(n)\right]\mathbf{e}(n), \quad (5)$$

where $\mathbf{I}_p$ denotes a $p \times p$ identity matrix.

Taking (5) into account, it results that

$$\varepsilon_{l+1}(n)=\left[1-\mu_l(n)\right]e_{l+1}(n), \quad (6)$$

where the variables $\varepsilon_{l+1}(n)$ and $e_{l+1}(n)$ denote the (l + 1)-th elements of the vectors $\boldsymbol{\varepsilon}(n)$ and $\mathbf{e}(n)$, with $l=0,1,\ldots,p-1$.

We propose to adjust the step size according to the square of the square of the time-averaged estimation of the autocorrelation of $\varepsilon_{l+1}(n)$ and $e_{l+1}(n)$. Therefore, the goal is to find an expression for the step-size parameter $\mu_l(n)$ such that

$$E\left\{\varepsilon_{l+1}(n)e_{l+1}(n)\right\}=E\left\{v^2(n-l)\right\} \quad (7)$$

where $E\{\bullet\}$ denotes mathematical expectation.

This assumption is different from that used in deriving the nonparametric VSS-NLMS [7] and it was recently used in [8]. It results:

$$\left[1-\mu_l(n)\right]E\left\{e_{l+1}^2(n)\right\}=E\left\{v^2(n-l)\right\}. \quad (8)$$

By solving (8), we obtain

$$\mu_l(n)=1-\frac{E\left\{v^2(n-l)\right\}}{E\left\{e_{l+1}^2(n)\right\}}. \quad (9)$$

From a practical point of view, (9) has to be evaluated in terms of power estimates as (10)

$$\mu_l(n)=1-\frac{\hat{\sigma}_v^2(n-l)}{\hat{\sigma}_{e_{l+1}}^2(n)} \quad . \quad (10)$$

The variable in the denominator can be computed in a recursive manner, i.e.,

$$\hat{\sigma}_{e_{l+1}}^2(n)=\lambda\hat{\sigma}_{e_{l+1}}^2(n-1)+(1-\lambda)e_{l+1}^2(n), \quad (11)$$

where λ is a weighting factor chosen as $\lambda = 1 - 1/(KL)$, with $K > 1$; the initial value is $\hat{\sigma}_{e_{l+1}}^2(0)=0$.

In the single-talk case, the near-end signal consists only of the background noise, $w(n)$. Its power could be estimated as follows:

$$\mu_l(n)=1-\frac{\hat{\sigma}_w^2}{\hat{\sigma}_{e_{l+1}}^2(n)}. \quad (12)$$

For a value of the projection order p = 1, the non-parametric VSS-NLMS (NVS-NLMS) algorithm proposed in [8] is obtained. For p > 1, a NVSS-APA can be derived, by computing (12) for $l=0,1,\ldots,p-1$. However, the algorithm can have good performance in the double-talk case or under-modeling situations if the same approach as in [10] is used. If it is supposed as in [10] that the adaptive filter has converged to a certain degree, it can be considered that $E\left\{y^2(n)\right\}\cong E\left\{\hat{y}^2(n)\right\}$(this is a very strong assumptions for practical systems) where

$$\hat{y}(n)=\mathbf{x}^T(n)\hat{\mathbf{h}}(n-1), \quad (13)$$

Therefore, we have $E\left\{v^2(n)\right\}\cong E\left\{d^2(n)\right\}-E\left\{\hat{y}^2(n)\right\}$, or in terms of power estimates $\hat{\sigma}_v^2(n)\cong\hat{\sigma}_d^2(n)-\hat{\sigma}_{\hat{y}}^2(n)$, with $\hat{\sigma}_d^2(n)$ and $\hat{\sigma}_{\hat{y}}^2(n)$ computed recursively as follows

$$\hat{\sigma}_d^2(n)=\lambda\hat{\sigma}_d^2(n-1)+(1-\lambda)d^2(n), \quad (14)$$

$$\hat{\sigma}_{\hat{y}}^2(n)=\lambda\hat{\sigma}_{\hat{y}}^2(n-1)+(1-\lambda)\hat{y}^2(n), \quad (15)$$

and the initial values are $\hat{\sigma}_{\hat{y}}^2(0)=0$ and $\hat{\sigma}_d^2(0)=0$.

Based on these findings, (11) can be rewritten as

$$\mu_l(n)=1-\frac{\hat{\sigma}_d^2(n-l)-\hat{\sigma}_{\hat{y}}^2(n-l)}{\hat{\sigma}_{e_{l+1}}^2(n)}, \quad (16)$$

for $l=0,1,\ldots,p-1$.

For a practical issue, a very small positive number ξ should be added to the denominator in (16) to avoid division by zero. Also, as shown in [17], a value of the step-size between 0 and 1 is preferable over the one between 1 and 2. Using the same assumptions from [10], the final step-sizes formula is written as

$$\mu_l(n)=\min\left(\left|1-\frac{\left|\hat{\sigma}_d^2(n-l)-\hat{\sigma}_{\hat{y}}^2(n-l)\right|}{\xi+\hat{\sigma}_{e_{l+1}}^2(n)}\right|,1\right), \quad (17)$$

for $l=0,1,\ldots,p-1$.

Summarizing, the proposed NVSS-APA equations are (1), (13), (14), (15), (11), (17), (3) and (2).

The NVSS-APA has $3p$ + 6 multiplication operations, p divisions, $4p$ + 2 additions and p comparisons more than APA. The step size formula for VSS-APA [10] is

$$\mu_l(n)=\left|1-\frac{\sqrt{\left|\hat{\sigma}_d^2(n-l)-\hat{\sigma}_{\hat{y}}^2(n-l)\right|}}{\xi+\hat{\sigma}_{e_{l+1}}(n)}\right|, \quad (18)$$

The equations of VSS-APA are (1), (13), (14), (15), (11), (18), (3) and (2). It can also be noticed that the step-size updating formula of NVSS-APA (17) doesn't have the p square-root operations of the step-size formula of VSS-APA therefore it is slightly less complex than VSS-APA. However, both VSS-APA and NVSS-APA require a matrix inversion that could be a very difficult operation, especially for high projection orders. Equation (2) can be rewritten as

$$\hat{\mathbf{h}}(n)=\hat{\mathbf{h}}(n-1)+\mathbf{X}(n)\mathbf{P}(n), \quad (19)$$

Where $\mathbf{P}$(n) is the solution of the following linear system

$$\left[\delta\mathbf{I}_p+\mathbf{X}^T(n)\mathbf{X}(n)\right]\cdot\mathbf{P}(n)=\boldsymbol{\mu}(n)\mathbf{e}(n), \quad (20)$$

We propose to iteratively solve (20) by using the DCD iterations [15]. The DCD algorithm updates a solution of a linear system of equations in directions of Euclidian coordinates in the cyclic order and with a step size that takes one of M_b (number of bits) predefined values corresponding to a binary representation bounded by an interval $[-H, H]$ [15-16]. The algorithm complexity is limited by N_u, the maximum number of "successful" iterations. The peak complexity of the DCD algorithm is $N(2N_u + M_b)$ shift-accumulation (SACs) operations [15]. Therefore, by replacing (2) with (19) and (20) in both NVSS-APA and VSS-APA, NVSS-APA-DCD and VSS-APA-DCD are obtained respectively.

III. SIMULATIONS

The simulations were performed in an AEC context. The length of the adaptive filter is set to 512 coefficients. The measured impulse response of the acoustic echo path is plotted in Fig. 1(a) (the sampling rate is 8 kHz); its entire length has 1024 coefficients. This length is truncated to the first 512 coefficients for an exact modeling case and to 1024 coefficients for the under-modeling case. The far-end signal, $x(n)$, is a speech sequence [Fig. 1(b)]. For the double-talk scenario, the near-end speech $u(n)$ (its power is equal to that of the far-end signal when they occur simultaneously) is plotted in Fig. 1(c). An independent white Gaussian noise signal $w(n)$ is added to the echo signal $y(n)$, with 30 dB signal-to-noise ratio (SNR). The weighting factor λ (for NVSS-APA and VSS-APA) is computed using K = 6 [10]. The value of the parameter in the denominator of (17) and (18) is $\xi = 10^{-8}$, and the regularization factor for all algorithms is $\delta = 50\sigma_x^2$. The DCD parameters were $H = 2^{-10}, M_b = 16$.

In Fig. 2, APA, VSS-APA, and NVSS-APA (all using the Geigel DTD [14]) performance in a double-talk situation was investigated. The Geigel DTD settings are chosen assuming a 6dB attenuation, i.e., the threshold is equal to 0.5 and the hangover time is set to 240 samples [14]. The near-end speech appears after 14 seconds from the debut of the adaptive process, for a period of 9.2 seconds. It can be noticed that both VSS based algorithms outperforms by far the APA.

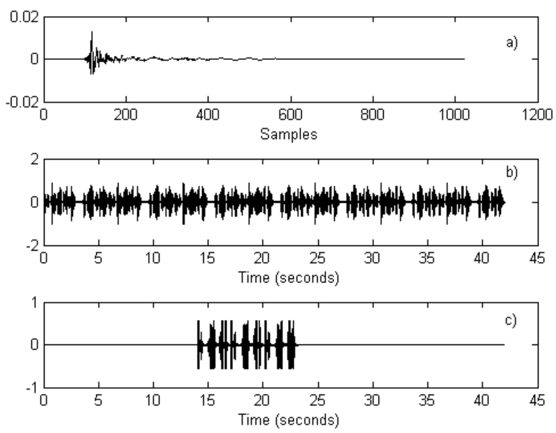

Figure 1. a) Measured room acoustic impulse response. b) Far-end speech signal used in the experiments. c) Near-end speech signal used in the last experiment.

Fig. 2 shows that NVSS-APA has the fastest initial convergence, but lower performance than VSS-APA in case of strong disturbances. The same behavior has been observed in simulations with white Gaussian noise or colored noise, and different projection orders. An explanation for this behavior can be found by examining Fig. 3 that shows the histograms of chosen step sizes (bins of width 0.1) for l=1 of VSS-APA and NVSS-APA respectively.

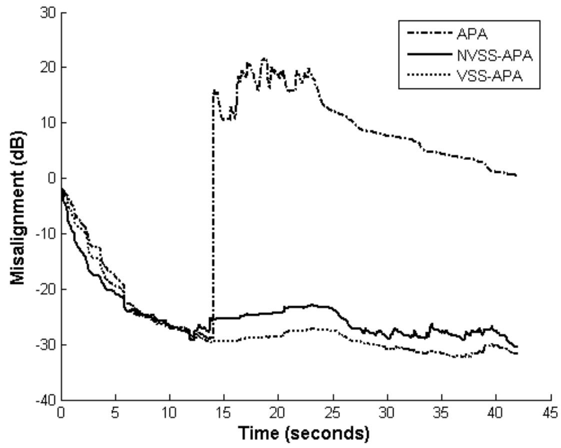

Figure 2. Misalignment of the APA with μ = 0.2, VSS-APA, and NVSS-APA for p = 2. The input signal is a speech signal. Double-talk case from Fig. 1c, with Geigel DTD and exact modeling.

It can be seen that, in general, NVSS-APA tends to choose higher step sizes than VSS-APA. By comparing (17) with (18) it is obvious that, if all the estimated variances are the same, the value subtracted from 1 in (18) is higher than that from (17) due to the use of the square root of a sub-unity value. These higher values of NVSS-APA step sizes lead to faster initial convergence, but also to possible higher misalignment.

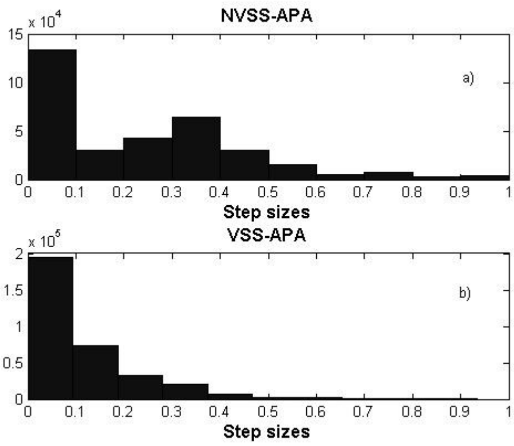

Figure 3. The histogram of the step sizes computed for Fig. 2 example (l=1 and bins of 0.1 width); a) NVSS-APA; b) VSS-APA.

Next, the DCD based algorithms (NVSS-APA-DCD and VSS-APA-DCD [18]) were simulated. Our simulations have confirmed previous results of using DCD [16]. If at least 16 DCD iterations are used NVSS-APA-DCD and VSS-APA-DCD have almost identical performance as NVSS-APA, and VSS-APA respectively. Since DCD's number of SACs increases linearly with the number of iterations, it is desired to use a reduced parameter N_u. As shown in [15-16] a smaller N_u lead to reduced performance. Fig. 4 shows the performance of

NVSS-APA-DCD and VSS-APA-DCD (both with one DCD iteration) in a double-talk and under-modeling situation.

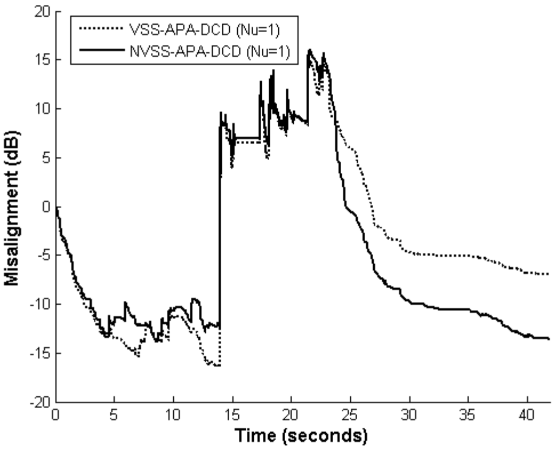

Figure 4. Misalignment of VSS-APA-DCD, and NVSS-APA-DCD for p = 2 (both algorithms using one DCD iteration), under-modeling case. The other conditions are from Fig. 2.

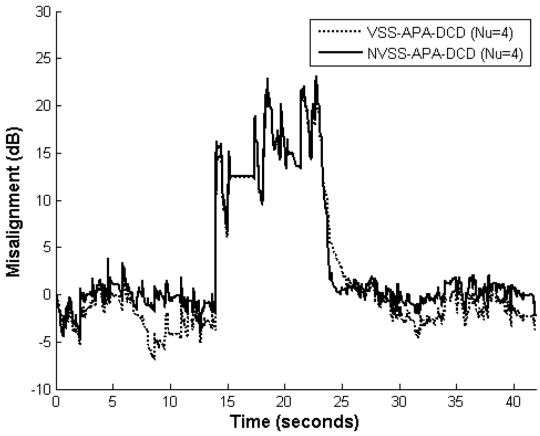

Figure 5. Misalignment of VSS-APA-DCD, and NVSS-APA-DCD for p = 8 (4 DCD iterations), exact-modeling case, SNR=10 dB.

Overall, in the investigated case, the new VSS equation (17) that favors higher step sizes lead to improved performance and recovery after strong disturbances with a reduced number of SAC's. Matrix inversion problems are more relevant for $p > 2$, but similar behavior was observed. However, both methods being power-based VSS algorithms have problems under unexpected conditions such as too low SNR. Figure 5 confirms that this problem for a SNR =10 dB, p = 8 and exact modeling case.

IV. CONCLUSIONS

In this paper, new non-parametric variable step size APAs were developed. The variable step-size formula of the proposed algorithm does not require any additional parameters from the acoustic environment. As compared to VSS-APA, it was found to have comparable performance in different simulations involving speech sequences and variants using DCD iterations were proposed.

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