Efficient multichannel filtered-x affine projection algorithm for active noise control

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An efficient multichannel affine projection algorithm for active noise control (ANC) systems is proposed. It is based on dichotomous co-ordinate descent iterations. It is shown that the proposed algorithm has a much lower complexity than the previously published multi-channel modified filtered-x affine projection algorithm for ANC, with similar convergence properties.

Introduction: Active noise control systems are being increasingly researched and developed [1]. In such systems, an adaptive controller is used to optimally cancel unwanted acoustic noise. The delay compensated modified filtered-x structure [2] for active noise control systems using FIR adaptive filtering is presented in Fig. 1. It is well known that the stochastic gradient descent algorithms have poor convergence speed, while the recursive-least-squares algorithms are too complex and often numerically unstable (see, e.g. [3] and references therein). The affine projection algorithms can provide a much improved convergence speed compared to stochastic gradient descent algorithms, without high increase of the computational load or the instability often found in recursive-least-squares algorithms. An affine projection algorithm for multichannel active noise control called the modified filtered-x affine projection algorithm (MFX-AP) has been presented in [3]. This algorithm is still too complex for practical applications. Therefore, simpler, fast affine projection (FAP) algorithms suitable for active noise control and based on some approximations of the original affine projection algorithm have been proposed recently (see, e.g. [3, 4]). All these algorithms need at least one inverse matrix computation, which is very complex for large matrices and prone to numerical instability. In this Letter, we propose the use of the dichotomous co-ordinate descent (DCD) iterations [5, 6] for solving the implicit linear system of the MFX-AP equations and avoid the inverse matrix computation. The resulting novel efficient algorithm is called the modified filtered-x dichotomous co-ordinate descent affine projection (MFX-DCDAP) algorithm. It includes the autocorrelation matrix updating procedure used in other fast affine projection algorithms for active noise control (ANC) [3, 4].

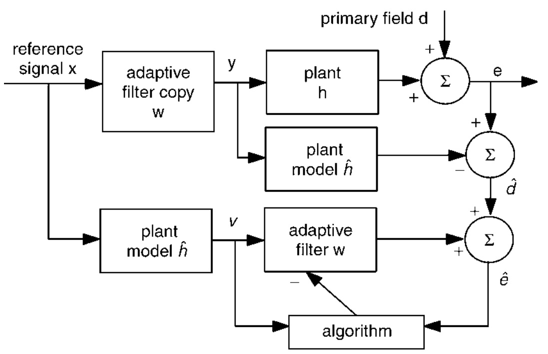


Fig. 1 *Delay compensated modified filtered-x structure for active noise control*

In the context of ANC systems, a multichannel feedforward system using an adaptive FIR filter with a modified filtered-x structure and with filter weights adapted with a classical affine projection (AP) algorithm can be described by the following equations (1)–(5) [3]:

$$y_j(n) = \sum_{i=1}^{I} \mathbf{w}_{i,j}^T(n)\mathbf{x}_i(n) \quad (1)$$

$$v_{i,j,k}(n) = \mathbf{h}_{j,k}^T \mathbf{x}_i'(n) \quad (2)$$

$$\hat{d}_k(n) = e_k(n) - \sum_{j=1}^{J} \mathbf{h}_{j,k}^T \mathbf{y}_j(n) \quad (3)$$

$$\hat{\mathbf{E}}^T(n) = \hat{\mathbf{D}}^T(n) + \mathbf{V}^T(n)\mathbf{w}(n) \quad (4)$$

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu\mathbf{V}(n)\left(\mathbf{V}^T(n)\mathbf{V}(n) + \delta\mathbf{I}\right)^{-1}\hat{\mathbf{E}}^T(n) \quad (5)$$

The variable n refers to the discrete time, I is the number of reference sensors, J represents the number of actuators, K is the number of error sensors, L is the length of the adaptive FIR filters, M is the length of (fixed) FIR filters modelling the plant and N is the projection order. The vectors $\mathbf{x}_i = [x_i(n), \ldots, x_i(n-L+1)]^T$ and $\mathbf{x}_i' = [x_i(n), \ldots, x_i(n-M+1)]^T$ consist of the last L and M samples of the reference signal $x_i(n)$, respectively. The vector $\mathbf{y}_j = [y_j(n), \ldots, y_j(n-M+1)]^T$ consists of the last M samples of the actuator signal $y_j(n)$. The samples of the filtered reference signal $v_{i,j,k}(n)$ are collected in the $IJ \times K$, $IJL \times K$ and $IJL \times KN$ matrices

$$\mathbf{V}_0(n) = \begin{bmatrix} v_{1,1,1}(n) & \cdots & v_{1,1,K}(n) \\ \vdots & \ddots & \vdots \\ v_{I,J,1}(n) & \cdots & v_{I,J,K}(n) \end{bmatrix}$$

$$\mathbf{V}_1(n) = [\mathbf{V}_0^T(n) \cdots \mathbf{V}_0^T(n-L+1)]^T \quad \text{and}$$

$$\mathbf{V}(n) = [\mathbf{V}_1(n) \cdots \mathbf{V}_1(n-N+1)]$$

The vectors $\hat{\mathbf{d}}(n) = [\hat{d}_1(n), \hat{d}_2(n), \ldots, \hat{d}_K(n)]$ and $\hat{\mathbf{e}}(n) = [\hat{e}_1(n), \hat{e}_2(n), \ldots, \hat{e}_K(n)]$ consist of estimates $\hat{d}_k(n)$ of the primary sound field $d_k(n)$ and of alternative error signals samples $\hat{e}_k(n)$, both computed in delay-compensated modified filtered-x structures. The vectors $\hat{\mathbf{D}}(n) = [\hat{\mathbf{d}}(n), \hat{\mathbf{d}}(n-1), \ldots, \hat{\mathbf{d}}(n-N+1)]$ and $\hat{\mathbf{E}}(n) = [\hat{\mathbf{e}}(n), \hat{\mathbf{e}}(n-1), \ldots, \hat{\mathbf{e}}(n-N+1)]$ have both $1 \times KN$ size. The vector $\mathbf{h}_{j,k} = [h_{j,k,1}, \ldots, h_{j,k,M}]^T$ consists of taps $h_{j,k,m}$ of the fixed FIR filter modelling the plant between signals $y_j(n)$ and $e_k(n)$. The $IJL \times 1$ vector $\mathbf{w}(n) = [[w_{1,1,1}(n) \cdots w_{I,J,1}(n)] \cdots [w_{1,1,L}(n) \cdots w_{I,J,L}(n)]]^T$ consists of the coefficients from all the adaptive FIR filters linking the signals $x_i(n)$ and $y_j(n)$. Finally, $e_k(n)$ is the kth error sensor signal, μ is a normalised convergence gain $0 \le \mu \le 1$, $\mathbf{I}$ is an identity matrix of size $KN \times KN$ and δ is the regularisation factor.

If we note by $\mathbf{R}(n)$ the autocorrelation matrix and by $\mathbf{P}(n)$

$$\mathbf{P}(n) = (\mathbf{V}^T(n)\mathbf{V}(n) + \delta\mathbf{I})^{-1}\hat{\mathbf{E}}^T(n) = (\mathbf{R}(n) + \delta\mathbf{I})^{-1}\hat{\mathbf{E}}^T(n) \quad (6)$$

Equation (5) becomes:

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu\mathbf{V}(n)\mathbf{P}(n) \quad (7)$$

We propose use of the DCD method for solving the linear system $(\mathbf{R}(n) + \delta\mathbf{I})\mathbf{P}(n) = \hat{\mathbf{E}}^T(n)$. The DCD algorithm is based on binary representation of elements of the solution vector with M_b bits within an amplitude range $[-H, H]$. The iterative approximation of the solution vector $\mathbf{P}(n)$ starts by updating the most significant bit of its elements and proceeds to less significant bits. If a bit update happens, the iteration is called 'successful', and the vector $\hat{\mathbf{E}}^T(n)$ is updated. The parameter N_{upd} represents the maximum number of 'successful' iterations. More details about the DCD algorithm can be found in [5, 6]. If H is a power of two the DCD algorithm is implemented only with additions and bit shifts operations [5]. Thus, the DCD algorithm can be implemented without explicit multiplications and divisions. The code describing the dichotomous co-ordinate descent algorithm can be found in [6]. The peak complexity of the DCD algorithm for given M_b and N_{upd}, is $N(2N_{upd} + M_b)$ shift-accumulation (SACs) operations.

The numerical complexities of the considered algorithms are measured by the number of multiplications per algorithm iteration [3]:

$$C_{MFX-DCDAP} = IJK(M + 2L + 2KN) + IJL + JKM \quad (8)$$

$$C_{MFX-AP} = IJL(2 + 2KN + K^2N^2) + JKM(1 + I) + K^2N^2 + K^3N^3/2 \quad (9)$$

$$C_{MFX-LMS} = IJK(M + 2L) + IJL + JKM \quad (10)$$

Table 1: Comparison of computational load of MFX-DCDAP algorithm with other delay-compensated modified filtered-x algorithms for ANC

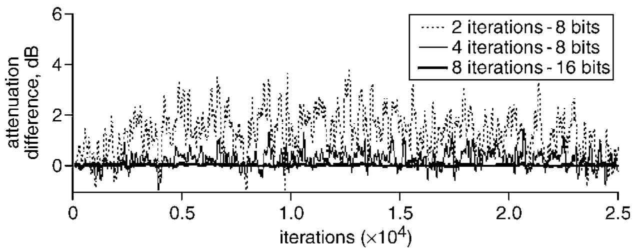
Algorithm for multichannel ANC, $L=100$, $M=64$, $N=5$	Multiplies per iteration for $I=1$, $J=1$, $K=1$	Multiplies per iteration for $I=1$, $J=3$, $K=2$
MFX-LMS	428	2268
MFX-DCDAP	438	2388
MFX-AP	3916	37 968

Table 1 evaluates the complexity of the considered algorithms for both monochannel and multichannel cases. It can be seen that the complexity of the MFX-DCDAP is much lower than that of MFX-AP, and only slightly more complex than MFX-LMS.

Results: The MFX-DCDAP algorithm was simulated and compared to the previously published multichannel modified filtered-x LMS algorithm (MFX-LMS) and the multichannel modified filtered-x affine projection algorithm (MFX-AP) [3]. We used in our simulation $I = 1$, $J = 3$, $K = 2$ and the reference signal was a white noise with zero mean and variance one. The simulations were performed with acoustic transfer functions experimentally measured in a duct. The impulse responses used for the multichannel acoustic plant had 64 samples each ($M = 64$), while the adaptive filters had 100 coefficients each ($L = 100$). For all the affine projection algorithms, a value of 0.9 was used for the step size μ and the regularisation factors were $\delta = \delta_2 = 2 \times 10^3$. The step size μ for the MFX-LMS algorithm was 2×10^{-5} and the parameter H of the DCD algorithm was set to $1/128$. The convergence performances have been averaged over 100 simulations. The performance of the algorithms was measured by

$$\text{Attenuation (dB)} = 10 \cdot \log_{10} \frac{\sum_k E[e_k^2(n)]}{\sum_k E[d_k^2(n)]} \qquad (11)$$

Fig. 2*a* shows that the implementation using eight DCD iterations and 16 bits provides almost identical performance with the method using the ideal inverse matrix. In this case the theoretical peak complexity is 160 SACs. Also, it can be seen in Fig. 2*a* that if an average loss of about 0.5 dB is allowed, the number of bits can be reduced to four and the number of DCD iterations to eight. Therefore, in this case, the peak DCD complexity is 80 SACs. However, the average DCD complexity is lower than the theoretical peak complexity in both cases (90 and 35, respectively). The DCD part increases the number of additions, but has no divisions or multiplications. Therefore, $N_{upd} = 8$ and $M_b = 16$ were used in the following simulation of the MFX-DCDAP algorithm. Fig. 2*b* compares the performance of the considered algorithms, with ideal plant models, for a multichannel ANC system, obtained from Matlab™ implementations of the algorithms. It can be seen that the MFX-DCDAP algorithm has almost identical performance with the previously published MFX-AP algorithm. As expected, the convergence performance of the affine projection algorithms is better than that of the LMS-based algorithm.



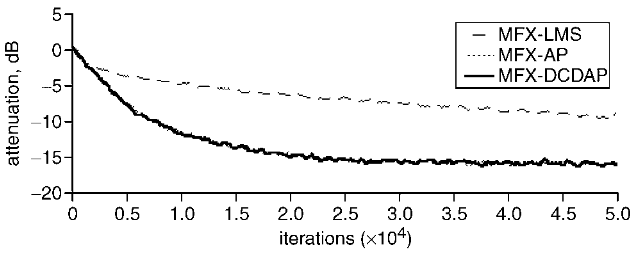


Fig. 2 *Attenuation difference over 25 000 iterations between convergence curves of algorithm using ideal matrix inverse and algorithm using different numbers of DCD iterations and bits and convergence curves of multichannel delay-compensated modified filtered-x algorithms for ANC, with ideal plant models*

$I = 1$, $J = 3$, $K = 2$, $L = 100$, $M = 64$, $N = 5$
a Attenuation difference
b Convergence curves

Conclusions: The modified filtered-x dichotomous co-ordinate descent affine projection (MFX-DCDAP) algorithm was introduced for practical active noise control systems using FIR adaptive filtering and was compared with the previously published MFX-AP and MFX-LMS algorithms. It was shown that MFX-DCDAP algorithm obtains almost identical performance with the much complex MFX-AP algorithm. Also, it was shown that it provides a significant improvement of convergence speed over the MFX-LMS algorithm, it being only slightly more complex than the latter. Therefore, it is a good candidate for practical real-time implementations.

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