A Recursive Least Square algorithm for Active Noise Control based on the Gauss-Seidel Method

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Abstract— **In this paper, a new recursive least-squares algorithm (RLS) for multichannel active noise control using the Gauss-Seidel method is proposed. It is shown that the proposed algorithm has a much lower complexity than the previously published modified filtered-x RLS algorithm, with similar convergence property and good numerical stability. Its logarithmic number system implementation is also presented.**

I. INTRODUCTION

Active noise control (ANC) systems are being increasingly researched and developed [1]. The delay compensated or modified filtered-x structure for active noise control systems using FIR adaptive filtering was introduced in [2], and it is presented in Fig. 1. The structure in Fig. 1 eliminates the plant delay by computing an estimate of the primary field signals, which are unaffected by the changes of the adaptive FIR filter coefficients.

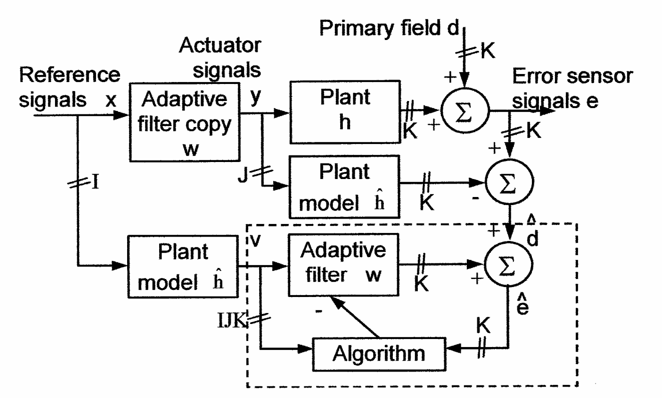

Figure 1. A delay compensated or modified filtered-x structure for active noise control.

A delay compensated or modified filtered-x structure for active noise control. For ANC using adaptive FIR filters, the multi-channel filtered-x least-mean-square (FX-LMS) algorithm [1-2] is the most commonly used algorithm. The drawback of the FX-LMS algorithm is the slow convergence speed, especially for broadband multi-channel systems. Although it converges faster than the FX-LMS algorithm, the delay compensated or modified filtered-x LMS algorithm (MFX-LMS) [2-3] also suffers from the same slow convergence problem, especially for multi-channel systems. The RLS based algorithms have a faster convergence speed. However, their complexity is much higher. Also, they often are affected by numerical instability [3]. Yet, the additional computational cost or the potential numerical instability in some of the proposed RLS algorithms for ANC can prevent the use of those algorithms for some applications.

In this paper, we use the formulation of the RLS problem in terms of a sequence of auxiliary normal equations with respect to increments of the filter weights [4]. This approach was applied to the exponentially weighted case and a new structure of the RLS algorithm was derived. In Section II, we propose the adaptation of the transversal Exponential RLS (ERLS) algorithm to multi-channel active noise control systems. We use the Gauss-Seidel method for solving the auxiliary linear system instead of the dichotomous coordinate descent algorithm [4]. Therefore a new algorithm called the multi-channel delay-compensated modified filtered-x Gauss-Seidel Exponential Recursive Least Square algorithm (MFX-GS-ERLS) is proposed. In Section III, we describe the logarithmic number system (LNS) implementation of the proposed algorithm [5-6]. Section IV presents simulation results of the proposed algorithm. A comparison of the numerical complexity of the proposed algorithm with the classical RLS algorithm is presented in Section V. Section VI concludes this work.

II. MULTICHANNEL MODIFIED FILTERED-X GAUSS SEIDEL EXPONENTIAL RECURSIVE LEAST SQUARE ALGORITHM

To describe the multi-channel delay-compensated Modified Filtered-X Gauss-Seidel Exponential Recursive Least Square algorithm (or MFX-GS-ERLS), the following notations are defined: I is the number of reference sensors in an ANC system; J is the number of actuators in an ANC system; K is the number of error sensors in an ANC system; L is the length of the adaptive FIR filters; M is the length of (fixed) FIR filters modeling the plant in an ANC system

$x_i(n)$ value at time n of the i^{th} reference signal

$y_j(n)$ value at time n of the j^{th} actuator signal

$d_k(n)$ value at time n of the primary sound field at the k^{th} error sensor

$e_k(n)$ value at time n of the k^{th} error sensor

$\hat{d}_k(n)$ estimate of $d_k(n)$, computed in delay-compensated or modified filtered-x structures

$\hat{e}_k(n)$ value at time n of an alternative error signal for the k^{th} sensor, computed in delay-compensated or modified filtered-x structures

$h_{j,k,m}$ value of the m^{th} coefficient in the (fixed) FIR filter modeling the plant between $y_j(n)$ and $e_k(n)$

$v_{i,j,k}(n)$ value at time n of the filtered reference signal

$w_{i,j,l}(n)$ value at time n of the l^{th} coefficient in the adaptive FIR filter linking $x_i(n)$ and $y_j(n)$

$\mathbf{R}(n)$ $IJL \times IJL$ auto-correlation matrix. $\mathbf{R}(n)$ is initialized as an identity matrix multiplied by the regularization factor δ.

$\mathbf{R}(n) = \begin{bmatrix} \overline{\mathbf{r}}(n) & \mathbf{r}^T(n) \\ \mathbf{r}(n) & \overline{\mathbf{R}}(n-1) \end{bmatrix}$, where $\overline{\mathbf{R}}(n)$ is the top left $IJ(L-1) \times IJ(L-1)$ values of $\mathbf{R}(n)$

$\mathbf{r}(n)$ correlation matrix of size $IJ(L-1) \times IJ$, initialized with zero values

$\overline{\mathbf{r}}(n)$ correlation matrix of size $IJ \times IJ$, initialized with zero values

$\mathbf{a}(n)$, $\mathbf{C}(n)$ and $\overline{\mathbf{w}}(n)$ are $IJL \times 1$ vectors used in solving the auxiliary equations.

$\boldsymbol{h}_{j,k} = \left[h_{j,k,1},\ h_{j,k,2},\ \cdots\ h_{j,k,M} \right]^T$ (size $M \times 1$)

$\mathbf{x}_i(n) = \left[x_i(n),\ x_i(n-1),\ \cdots\ x_i(n-L+1) \right]^T$ (size $L \times 1$)

$\mathbf{x}'_i(n) = \left[x_i(n),\ x_i(n-1),\ \cdots\ x_i(n-M+1) \right]^T$ (size $M \times 1$)

$\mathbf{y}_j(n) = \left[y_j(n),\ y_j(n-1),\ \cdots\ y_j(n-M+1) \right]^T$ (size $M \times 1$)

$\hat{\mathbf{d}}(n) = \left[\hat{d}_1(n), \hat{d}_2(n), \cdots \hat{d}_K(n) \right]$ (size $1 \times K$)

$\hat{\mathbf{e}}(n) = \left[\hat{e}_1(n), \hat{e}_2(n), \cdots \hat{e}_K(n) \right]$ (size $1 \times K$)

$\mathbf{V}_0(n) = \begin{bmatrix} v_{1,1,1}(n) & \cdots & v_{1,1,K}(n) \\ \vdots & \ddots & \vdots \\ v_{I,J,1}(n) & \cdots & v_{I,J,K}(n) \end{bmatrix}$ (size $IJ \times K$)

$\mathbf{V}(n) = \left[\mathbf{V}_0^T(n) \cdots \mathbf{V}_0^T(n-L+1) \right]^T = \left[\mathbf{V}_0^T(n)\ \mathbf{V}_r^T(n) \right]^T$ (size $IJL \times K$)

$\mathbf{w}(n) = \left[\left[w_{1,1,1}(n)\ \ldots\ w_{I,J,1}(n) \right]\ \cdots\ \left[w_{1,1,L}(n)\ \cdots\ w_{I,J,L}(n) \right] \right]^T$ (size $IJL \times 1$)

We have $\mathbf{C}(n) = \lambda \mathbf{a}(n) - \mathbf{V}(n) \hat{\mathbf{e}}^T(n)$ and $\mathbf{a}(n) = \mathbf{C}(n) - \mathbf{R}(n) \overline{\mathbf{w}}(n)$. We assume that $\mathbf{a}(n)$ is a null vector (corresponds to the ideal solution). We solve the linear system $\mathbf{R}(n) \overline{\mathbf{w}}(n) = \mathbf{V}(n) \hat{\mathbf{e}}^T(n)$ with one Gauss-Seidel iteration [7]. An important complexity reduction is obtained and the simulations have shown that the stability of the proposed algorithm is improved as well.

The multi-channel filtered-x Gauss-Seidel exponential recursive least square algorithm is defined by the following equations:

$$y_j(n) = \sum_{i=1}^{I} \mathbf{w}_{i,j}^T(n) \mathbf{x}_i(n) \tag{1}$$

$$v_{i,j,k}(n) = \mathbf{h}_{j,k}^T \mathbf{x}'_i(n) \tag{2}$$

$$\hat{d}_k(n) = e(n) - \sum_{j=1}^{J} \mathbf{h}_{j,k}^T \mathbf{y}_j(n) \tag{3}$$

$$\hat{\mathbf{e}}^T(n) = \hat{\mathbf{d}}^T(n) + \mathbf{V}^T(n) \mathbf{w}(n) \tag{4}$$

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$$\bar{\mathbf{r}}(n)=\lambda\bar{\mathbf{r}}(n)+\mathbf{V}_0(n)\mathbf{V}_0^T(n) \tag{5}$$

$$\mathbf{r}(n)=\lambda\mathbf{r}(n)+\mathbf{V}_0(n)\mathbf{V}_r^T(n) \tag{6}$$

$$\mathbf{R}(n)\overline{\mathbf{w}}(n)=\mathbf{V}(n)\hat{\mathbf{e}}^T(n) \tag{7}$$

$$\mathbf{w}(n+1)=\mathbf{w}(n)-\overline{\mathbf{w}}(n) \tag{8}$$

III. THE LOGARITHMIC NUMBER SYSTEMS

As an alternative to floating-point, the logarithmic number system offers the speed advantages when implementing algorithms with many multiplication, division and square-root operations. These advantages are, however, offset by the problem of performing logarithmic addition and subtraction. The LNS arithmetic operations are presented in Table 1.

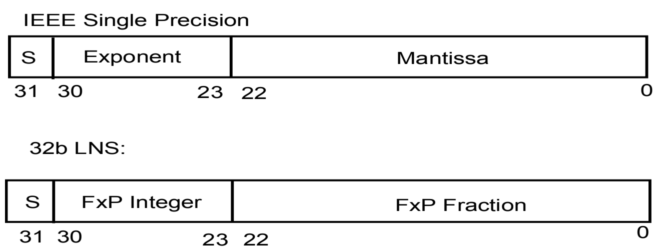

Figure 2. IEEE standard single precision floating point representation and the 32-bit LNS format.

TABLE I. LNS ARITHMETIC OPERATIONS.

x + y	ADD	Lz = Lx + log(1+2^(Ly-Lx)), Sz depends on sizes of x,y
x - y	SUB	Lz = Lx + log(1-2^(Ly-Lx)), Sz depends on sizes of x,y
x * y	MUL	Lz = Lx + Ly, Sz = Sx OR Sy
x / y	DIV	Lz = Lx - Ly, Sz = Sx OR Sy
x^2	SQU	Lx << 1, Sz = Sx
x^0.5	SQRT	Lx >> 1, Sz = Sx
x^-1	RECIP	Lz = Lx, Sz = -Sx
x^-0.5	RSQRT	Lz = Lx >> 1, Sz = -Sx

The 32-bit floating-point representation consists of a sign, 8-bit biased exponent, and 23-bit mantissa. The LNS format is similar in structure (see Fig. 1). The 'S' bit again indicates the sign of the real value represented, with the remaining bits forming a 31-bit fixed point word in which the size of the value is encoded as its base-2 logarithm in 2's complement format. The chosen format compares favorably against its floating-point counterpart, having greater range and slightly smaller representation error [5]. A 20-bit LNS format is similar. It maintains the same range as the 32-bit, but has precision reduced to 11 fractional bits. More details about the logarithmic number system and some of its applications are available in [5-6].

IV. SIMULATIONS (22)

The new MFX-GS-ERLS algorithm was simulated and compared to the previously published multi-channel modified filtered-x LMS algorithm (MFX-LMS, [3]), multi-channel modified filtered-x affine projection algorithm (MFX-AP, [3]) and the multi-channel modified filtered-x RLS algorithm (MFX-RLS, [3]). We used in our simulation $I=1, J=3, K=3, \lambda=0.995$ and the reference signal was a white noise with zero mean and variance one. The simulations were performed with acoustic transfer functions experimentally measured in a duct. The impulse responses used for the multi-channel acoustic plant had 64 samples each ($M=64$), while the adaptive filters had 100 coefficients each ($L=100$). The step size μ for the MFX-LMS algorithm was $2\cdot 10^{-5}$. The performance of the algorithms was measured by

$$Attenuation(dB)=10\cdot\log_{10}\frac{\sum_k E\left[e_k^2(n)\right]}{\sum_k E\left[d_k^2(n)\right]} \tag{9}$$

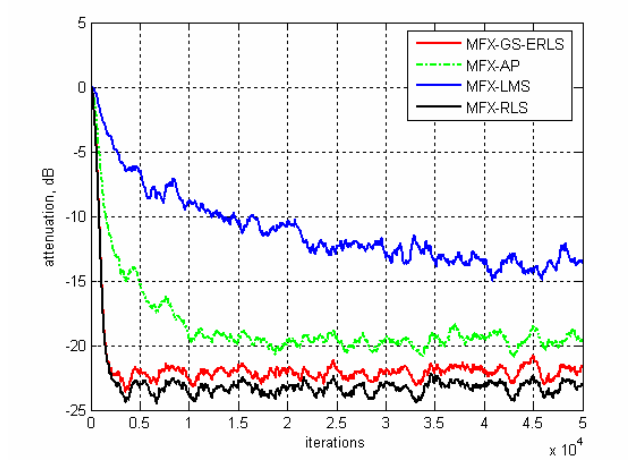

Figure 3. Convergence curves of multichannel delay-compensated modified filtered-x algorithms for ANC with ideal plant models

Fig. 3 compares the performance of the selected algorithms with ideal plant models, for a multi-channel ANC system obtained from Matlab™ implementations of the algorithms (double precision 64 bits floating point format). It can be seen that the MFX-GS-ERLS and MFX-RLS algorithms have the same convergence speed. Also, it can be seen that the MFX-GS-ERLS algorithm has a small loss in attenuation due to the approximation used in deriving the algorithm. As expected, their convergence performance is better than that of the MFX-LMS and MFX-AP algorithm (a projection order of N =5 was used).

An accurate standard for comparison of the outputs of the LNS implementation of MFX-GS-ERLS and MFX-RLS was obtained by presenting the input data to their double precision versions and compute the accumulated absolute sum of errors of the 20-bit or 32-bit LNS outputs.

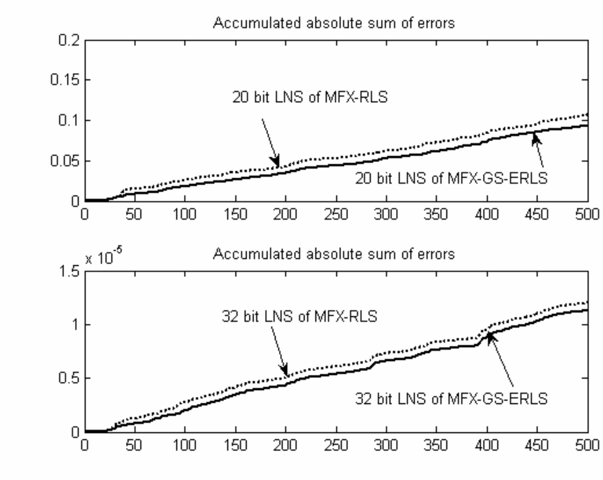

Figure 4. (upper) The acummulated absolute sum of errors for 20-bit LNS implementations of the investigated algorithms; (lower) The acummulated absolute sum of errors for 32-bit LNS implementations of the MFX-GS-ERLS algorithm.

The accumulated absolute sum of errors for the 32-bit LNS implementation of MFX-GS-ERLS (solid line in Fig. 4 - lower) is smaller than that of the 32-bit LNS implementation of MFX-RLS algorithm (dotted line). The 32-bit LNS implementation was stable in our simulations and the results were virtually identical to the double precision results. Similar results were also reported in [6] using another class of algorithms. As expected, the accumulated absolute sum of errors for the 20-bit LNS implementation is much higher than that of the 32-bit LNS implementation. However, few dB losses in attenuation and signs of instability are possible if the 20-bit versions are used.

V. Computational Complexity

The computational complexity of the proposed algorithms was estimated by the number of multiplications required per iteration. Matrix inversions were assumed to be performed with standard LU decomposition that requires $O\{X^3/2\}$ multiplications, where X is the size of a square matrix. As a reference for comparison, the number of multiplications per MFX-RLS algorithm iteration is [3]:

$$L^2\left(2I^2J^2K + I^2J^2\right) + L\left(2IJK^2 + 3IJK + IJ\right) + M\left(IJK + JK\right) + K^2 + K + K^3/2 \qquad (10)$$

The number of multiplications per MFX-GS-ERLS algorithm iteration is:

$$IJL\left(IJL + IJK^2 + IJ + 2K + 2\right) + JKM(I+1) \qquad (11)$$

TABLE II. Comparison of computational load of MFX-GS-ERLS algorithm with other delay-compensated modified filtered-x algorithms for ANC

Algorithm for multichannel ANC, L=100, M=64, N=5	Multiplies per iteration for I=1, J=1, K=1	Multiplies per iteration for I=1, J=3, K=2
MFX-LMS	428	2268
MFX-AP	3916	37968
MFX-GS-ERLS	**10728**	**97068**
MFX-RLS	30730	455278

It can be seen from Table II that the MFX-RLS algorithm has many more multiplications per iteration than the MFX-GS-ERLS algorithm. The reduction in complexity depends on the values of the parameters (*I, J, K, L, M)*. For example, for the mono-channel case the reduction is about 65%, while it is about 80 % for the investigated parameters of the multi-channel case.

VI. Conclusions

The MFX-GS-ERLS algorithm was introduced for multi-channel ANC. It is shown that it has similar converge speed obtained with a significant lower complexity than that of the MFX-RLS algorithm. Also, the MFX-GS-ERLS showed better numerical properties than the original MFX-RLS algorithms in logarithmic number system simulations.

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