inter.noise 2010
noise and sustainability
13-16 June 2010 . Lisbon . Portugal
www.internoise2010.org

New filtered-x recursive least square algorithm for active noise control

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Abstract

A new multichannel filtered-x recursive least square algorithm for active noise control systems is proposed. It is shown that the use of the filtered-x structure, instead of the commonly used modified filtered-x structure lead to a more efficient implementation and similar convergence performance and stability. The paper is also focused on examining the benefits of auxiliary normal equations solving methods needed by the formulation of the RLS problem. The behavior of the proposed algorithm is investigated in cases of ideal and non-ideal acoustic plants.

Keywords: filtered-x structures, active noise control, recursive least squares

1 Introduction

During last years many adaptive algorithms have been proposed for multichannel active noise control (ANC) applications. In ANC systems, an adaptive controller is used to optimally cancel unwanted acoustic noise [1], [2].

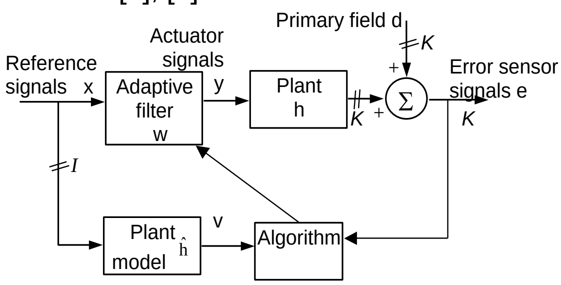

Fig. 1. Block diagram of the filtered-x structure for active noise control.

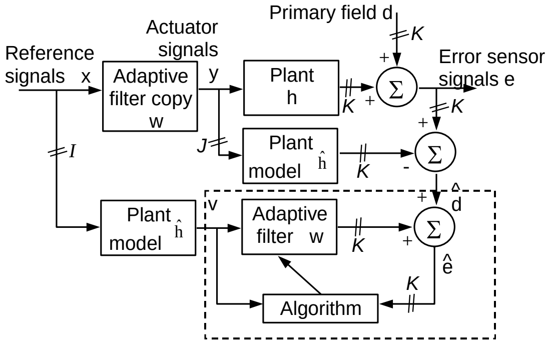

Fig. 2. Block diagram of the delay compensated (or modified filtered-x) structure for active noise control.

The filtered-x (Fig. 1) and modified filtered-x (Fig. 2) are commonly used structures for active noise control systems [3-11]. Multichannel versions of the filtered-x least-mean-square (FX-LMS) and modified FX-LMS (MFX-LMS) algorithms are the benchmarks to which most adaptive filtering algorithms are compared, because they are widely used [1-4].

Affine projection (AP) adaptive algorithms and its numerous fast versions are also known to be efficient when using in ANC systems (e.g., see [4-8] and references therein). However, they possess a slower convergence compared to RLS algorithms.

It is well known that recursive-least-squares (RLS) algorithms have much faster convergence than previously mentioned algorithms, but they are too complex and often numerically unstable (e.g., see [9-12] and references therein). Extensions of stable realizations of RLS algorithms such as the inverse QR-RLS, the QR decomposition least-squares-lattice (QRD-LSL) algorithms for specific ANC systems were presented in [10]. However, they are too complex for practical implementations. In [13] a new formulation of the RLS problem in terms of a sequence of auxiliary normal equations with respect to increments of the filter weights has been proposed. This approach was applied to the exponentially weighted RLS algorithm and a new structure of the transversal exponential RLS algorithm was derived [13].

In this paper, we adapt the structure of the transversal exponential RLS algorithm for the multichannel filtered-x structure and derive a novel efficient algorithm called the filtered-x Gauss-Seidel exponential recursive least squares (FX-GS-ERLS) algorithm. We compare it with the modified filtered-x version called MFX-GS-ERLS algorithm [14]. These algorithms are introduced in Section 2. The computational complexity of the proposed algorithm is evaluated and compared with the complexity of other algorithms in Section 3. Simulation results are presented in Section 4. Section 5 concludes this work.

2 A new exponential recursive least square algorithm for ANC

In order to describe the algorithms most of notations and definitions from [5] are used. The variable *n* refers to the discrete time, *I* is the number of reference sensors, *J* represents the number of actuators, *K* is the number of error sensors, *L* is the length of adaptive FIR filters, *M* is the length of fixed FIR filters modeling the plant.

The vectors $\mathbf{x}_i = \left[x_i(n),...,x_i(n-L+1)\right]^T$ and $\mathbf{x}_i^{'} = [x_i(n),...,x_i(n-M+1)]^T$ consist of the last *L* and *M* samples of the reference signal $x_i(n)$, respectively. The vector $\mathbf{y}_j = [y_j(n),...,y_j(n-M+1)]^T$ consists of the last *M* samples of the actuator signal $y_j(n)$. The samples of the filtered reference signal $v_{i,j,k}(n)$ are collected in the $IJ \times K$, and $IJL \times K$ matrices

$\mathbf{V}_0(n)=\begin{bmatrix} v_{1,1,1}(n)\ldots v_{1,1,K}(n) \\ \ldots. \qquad \ldots\ldots \\ v_{I,J,1}(n)\ldots v_{I,J,K}(n)\end{bmatrix}$ and $\mathbf{V}(n)=\left[\mathbf{V}_0^T(n)\cdots\mathbf{V}_0^T(n-L+1)\right]^T=\left[\mathbf{V}_0^T(n)\ \mathbf{V}_r^T(n)\right]^T$, respectively. The vectors $\hat{\mathbf{d}}(n)=\left[\hat{d}_1(n),\hat{d}_2(n),\ldots,\hat{d}_K(n)\right]$ and $\hat{\mathbf{e}}(n)=\left[\hat{e}_1(n),\hat{e}_2(n),\ldots,\hat{e}_K(n)\right]$ consist of estimates $\hat{d}_k(n)$ of the primary sound field $d_k(n)$ and of alternative error signal samples $\hat{e}_k(n)$, both computed in delay-compensated modified filtered-x structures. For the filtered-x structure, the multichannel error vector is $\mathbf{e}(n)=\left[e_1(n),e_2(n),\ldots,e_K(n)\right]$. The vectors $\mathbf{h}_{j,k}=\left[h_{j,k,1},\ldots,h_{j,k,M}\right]^T$ consist of taps $h_{j,k,m}$ of the fixed FIR filter modelling the plant between signals $y_j(n)$ and $e_k(n)$. The $IJL\times 1$ vector $\mathbf{w}(n)=\left[\left[w_{1,1,1}(n)\ldots w_{I,J,1}(n)\right]\ldots\left[w_{1,1,L}(n)\ldots w_{I,J,L}(n)\right]\right]$ consists of the coefficients from all the adaptive FIR filters linking the signals $x_i(n)$ and $y_j(n)$ [5]. $\mathbf{R}(n)$ is a $IJL\times IJL$ auto-correlation matrix initialized as an identity matrix multiplied by a regularization factor δ. It is updated as $\mathbf{R}(n)=\begin{bmatrix}\bar{\mathbf{r}}(n) & \mathbf{r}^T(n)\\ \mathbf{r}(n) & \overline{\mathbf{R}}(n-1)\end{bmatrix}$, where $\overline{\mathbf{R}}(n)$ is the top left $IJ(L-1)\times IJ(L-1)$ elements of $\mathbf{R}(n)$, $\mathbf{r}(n)$ is a matrix of size $IJ(L-1)\times IJ$, initialized with zero values and $\bar{\mathbf{r}}(n)$ is a $IJ\times IJ$ matrix, also initialized with zero values. $e_k(n)$ is the *k*th error sensor signal, λ is the forgetting factor, and η is a gain scalar. Finally, $\mathbf{a}(n)$, $\mathbf{C}(n)$ and $\overline{\mathbf{w}}(n)$ are initially null $IJL\times 1$ vectors used in solving the auxiliary equations.

In the context of ANC systems, a multichannel feedforward system using an adaptive FIR filter with a filtered-x structure and with filter weights adapted with the ERLS algorithm can be described by the following equations:

$$y_j(n)=\sum_{i=1}^{I}\mathbf{w}_{i,j}^T(n)\mathbf{x}_i(n) \tag{1}$$

$$v_{i,j,k}(n)=\mathbf{h}_{j,k}^T\mathbf{x}_i'(n) \tag{2}$$

$$e_k(n)=d_k(n)+\sum_{j=1}^{J}\mathbf{h}_{j,k}^T\mathbf{y}_j(n) \tag{3}$$

$$\bar{\mathbf{r}}(n)=\lambda\bar{\mathbf{r}}(n-1)+\mathbf{V}_0(n)\mathbf{V}_0^T(n) \tag{4}$$

$$\mathbf{r}(n)=\lambda\mathbf{r}(n-1)+\mathbf{V}_0(n)\mathbf{V}_r^T(n) \tag{5}$$

$$\mathbf{C}(n)=\lambda\mathbf{a}(n-1)+\mathbf{V}(n)\mathbf{e}^T(n) \tag{6}$$

$$\mathbf{R}(n)=\begin{bmatrix}\bar{\mathbf{r}}(n) & \mathbf{r}^T(n)\\ \mathbf{r}(n) & \overline{\mathbf{R}}(n-1)\end{bmatrix} \tag{7}$$

$$\mathbf{R}(n)\overline{\mathbf{w}}(n)=\mathbf{C}(n) \tag{8}$$

(solved with one GS iteration)

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \eta\overline{\mathbf{w}}(n) \tag{9}$$

$$\mathbf{a}(n) = \mathbf{C}(n) - \mathbf{R}(n)\overline{\mathbf{w}}(n) \tag{10}$$

The gain scalar η is introduced in order to reduce the modifications of $\mathbf{w}(n)$ if there is a large delay in the plant. The matrices $\mathbf{r}(n)$ and $\bar{\mathbf{r}}(n)$ obtained in equations (4) and (5), respectively, allow the efficient update (7) of the matrix $\mathbf{R}(n)$. This update exploits the time-shift structure of the input data. The simulations have shown that one Gauss-Seidel iteration using $\overline{\mathbf{w}}(n-1)$ as an initial approximation in solving the Eq. 8 is approximating well the solution using the ideal inverse matrix. The consequence is that $\lambda\mathbf{a}(n-1) << \mathbf{V}(n)\mathbf{e}^T(n)$ and $\mathbf{C}(n) \cong \mathbf{V}(n)\mathbf{e}^T(n)$ [12]. Therefore, Eq. 6 can be replaced by

$$\mathbf{C}(n) = \mathbf{V}(n)\mathbf{e}^T(n) \tag{6'}$$

and only $\overline{\mathbf{w}}(n)$ is needed from the Gauss-Seidel method. This leads to a reduced numerical complexity with insignificant influence on the performance of the algorithm.

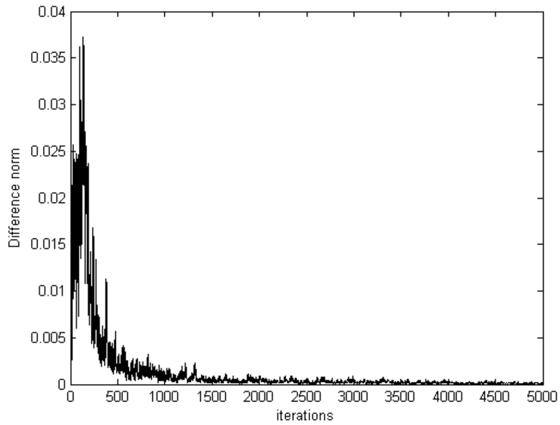

Fig. 3. The norm of the difference between the Gauss-Seidel computed $\overline{\mathbf{w}}(n)$ vectors when the $\mathbf{C}(n)$ vector is obtained using Eq. (6) and Eq. (6') respectively.

Fig. 3 shows the norm of the difference between the Gauss-Seidel computed $\overline{\mathbf{w}}(n)$ vectors when the $\mathbf{C}(n)$ vector is obtained using Eq. (6) and Eq. (6') respectively. It can be seen that this difference is very small and justify the approximation used in deriving the FX-GS-ERLS algorithm. This difference is initially higher at beginning and becoming much smaller after the algorithm convergence. Similar behavior is observed for both FX and MFX structures.
Therefore the proposed FX-GS-ERLS algorithm is described by the equations (1)-(5), (6'), (7)-(9).

The modified filtered-x GS-ERLS algorithm has the following three different equations for estimates of the primary sound field computed in delay-compensated modified filtered-x structure (Eq. 3a), the alternative error signal (Eq. 3b), and $\mathbf{C}(n)$ (Eq. 6a):

$$\hat{d}_k(n) = e_k(n) - \sum_{j=1}^{J} \mathbf{h}_{j,k}^T \mathbf{y}_j(n) \tag{3a}$$

$$\hat{\mathbf{e}}^T(n) = \hat{\mathbf{d}}^T(n) + \mathbf{V}^T(n)\mathbf{w}(n) \tag{3b}$$

$$\mathbf{C}(n) = \lambda \mathbf{a}(n-1) + \mathbf{V}(n)\hat{\mathbf{e}}^T(n) \tag{6a}$$

Similar numerical complexity approach as above is validated by the simulation results. Therefore, the Eq. (6a) can be replaced by Eq. (6') and only $\overline{\mathbf{w}}(n)$ is computed by the Gauss-Seidel method. The multichannel modified filtered-x MFX-GS-ERLS algorithm [14] for active noise control can be described by the following equations (1)-(2), followed by (3a) and (3b), (4)-(5), (6'), (7)-(9).

3 Computational complexity

The numerical complexity of the considered algorithms is measured by the number of multiplications ($M_{\text{algorithm}}$) per algorithm iteration. Matrix inversions were assumed to be performed with standard LU decomposition that requires $O(X^3/2)$ multiplications, where X is the size of a square matrix [6]. We investigated the numerical complexity of the MFX-LMS [9], MFX-GS-ERLS, FX-GS-ERLS, and modified filtered-x fast transversal filter (MFX-RLS) [10] algorithms:

$$M_{FX-LMS} = IJK(L+M) + IJL + K \tag{10}$$

$$M_{MFX-LMS} = IJK(2L+M) + IJL + JKM + K \tag{11}$$

$$M_{FX-GS-ERLS} = IJL(IJL + IJ + IJK + K + 2) + IJKM \tag{12}$$

$$M_{MFX-GS-ERLS} = IJL(IJL + IJ + IJK + 2K + 2) + JKM(I+1) \tag{13}$$

$$M_{FX-RLS} = L^2\left(2I^2J^2K + I^2J^2\right) + K^2 + K + K^3/2 + L\left(2IJK^2 + 2IJK + IJ\right) + MIJK \tag{14}$$

$$M_{MFX-RLS} = L^2\left(2I^2J^2K + I^2J^2\right) + K^2 + K + K^3/2 + L\left(2IJK^2 + 3IJK + IJ\right) + M\left(IJK + JK\right) \tag{15}$$

Fig. 4 shows the ratio of the number of multiplications of FX-RLS and FX-GS-ERLS (i.e., $M_{FX-RLS}/M_{FX-GS-ERLS}$) for different *L* values. It can be seen that the complexity reduction ratio is increasing for higher *I, J,* and *K* values. However, the complexity reduction ratio increase is slowing with increased L values for fixed *I, J* and *K* values. Similar conclusions can be obtained for the MFX-GS-ERLS algorithm.
Usually we have $L \gg \{I, J, K, N\}$ in practical implementations and therefore, in terms of multiplications, both FX-GS-ERLS and MFX-GS-ERLS algorithms are less complex than the FX-RLS/MFX-RLS algorithms.

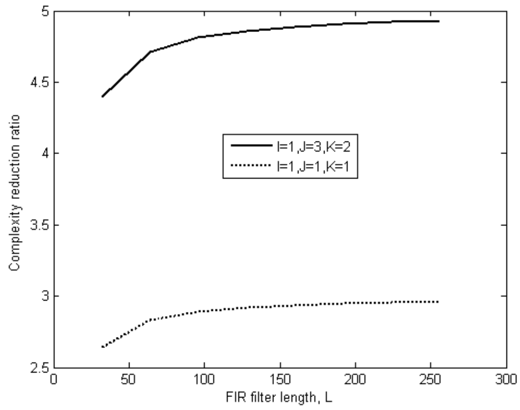

Fig. 4. The ratio of the multiplications of the FX-RLS and FX-GS-ERLS algorithms for a varying *L*, of the multichannel case ($I=1, J=3, K=2, M=64$) and monochannel case ($I=1, J=1, K=1, M=64$).

The least complex is the MFX-LMS algorithm and then, in the order of increasing the complexity, we have the FX-GS-ERLS, MFX-GS-ERLS, FX-RLS and MFX-RLS algorithms.
Table 1 shows the number of multiplications of the considered algorithms for both monochannel ($I=1, J=1, K=1$) and multichannel case ($I=1, J=3, K=2$). It can be seen that, in terms of multiplications, the complexity of the MFX-GS-ERLS and FX-GS-ERLS algorithms is significantly lower than that of the FX-RLS/MFX-RLS algorithm. For example, for the mono-channel case the reduction is about 65%, while it is about 80 % for the investigated parameters of the multichannel case.

Table 1. Comparison of the computational load of the FX-GS-ERLS and MFX-GS-ERLS algorithms with other multichannel delay-compensated modified filtered-x algorithms for ANC

Algorithm $M=64, L=128$	Multiplies per iteration $I=1, J=1, K=1$	Multiplies per iteration $I=1, J=3, K=2$
FX-LMS	321	1538
MFX-LMS	513	2690
FX-GS-ERLS	17088	152832
MFX-GS-ERLS	17280	153984
FX-RLS	49858	742666
MFX-RLS	50051	743818

4 Simulation results

The MFX-GS-ERLS and FX-GS-ERLS algorithms were simulated and their performance was compared to the previously published MFX-LMS and MFX-RLS algorithms. In our simulation, we used $I=1, J=3, K=2, \lambda=0.9995$ and the reference signal was a white noise with zero mean and variance one. The simulations are performed with acoustic transfer functions experimentally measured in a duct. The impulse responses used for the multichannel acoustic plant have 64 samples each ($M=64$), while the adaptive filters have 128 coefficients each ($L = 128$). The regularization factor was $\delta = 2 \cdot 10^3$ for the ideal plant and $\delta = 10^4$ for plant models with SNR of 10 dB. The step size μ for the MFX-LMS algorithm was $2 \cdot 10^{-5}$ and 10^{-6} for the FX-LMS algorithm. The performance of the algorithms was measured by

$$\text{Attenuation}\,(\text{dB}) = 10 \cdot \log_{10} \frac{\sum_k E\left[e_k^2(n)\right]}{\sum_k E\left[d_k^2(n)\right]} \tag{16}$$

Fig. 5 shows the absolute value of the attenuation difference between the convergence curves obtained with the direct solution and the Gauss-Seidel iterative solution using one or d two GS iterations. As expected, increasing the number of GS iterations leads to a closer performance to that obtained by using the linear system with an exact method (Fig. 5). It can be seen that the FX-GS-ERLS algorithm using only one GS iteration has almost identical performance with the more complex FX-RLS algorithm. The performance of the FX-GS-ERLS algorithm with one GS iteration is slightly worse than that obtained using two GS iterations.

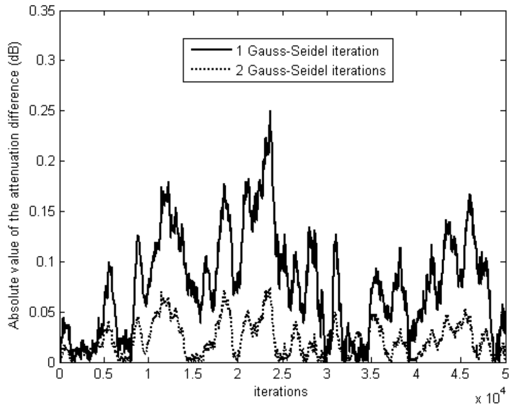

Fig. 5. The absolute value of the attenuation difference between the convergence curves obtained with the direct solution and the Gauss-Seidel iterative solution (one and two GS iterations).

One supplementary Gauss-Seidel iteration adds $I^2 J^2 L^2$ more multiplications to the numerical complexity of the algorithms. This is an important complexity increase because $L >> \{I, J\}$ and, therefore, one GS iteration leads to an excellent compromise between numerical complexity and convergence performance.
Figure 6 compares the performance of the selected algorithms, with ideal plant models, for a multichannel system ($I = 1, J = 3, K = 2$, $\eta = 1$). As shown in [3] a value of η=1 is an optimal value for the RLS algorithm, and the solution of the least-squares equations is computed at every iteration of the algorithm. It can be seen from Fig. 6 that, in case of ideal plant models, the performance of the MFX-GS-ERLS algorithm is very close to that of the MFX-RLS algorithm. Also, as it is already known, it shows that the convergence of the MFX-GS-ERLS is better than that of the MFX-LMS algorithm. Fig. 7 shows that the performance of the FX-GS-ERLS algorithm is very close to that of the FX-RLS algorithm or MFX-GS-ERLS algorithm (Fig. 6). Figures 8 and 9 shows the performance of the investigated algorithms when non-ideal plant models with a 10 dB SNR are used. The noisy plant models with 10 dB SNR accuracy were obtained as in [9].

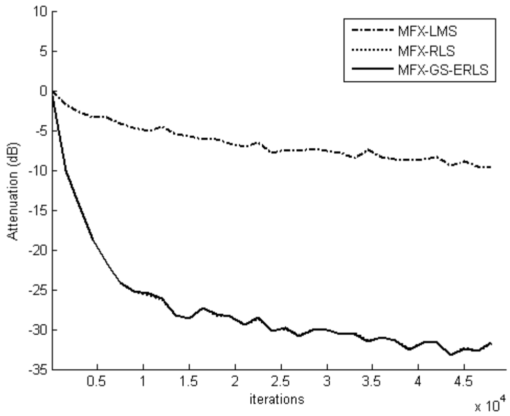

Fig. 6. Convergence of multichannel delay-compensated modified filtered-x algorithms for ANC, with ideal plant models ($I=1, J=3, K=2, L=128$, $M=64$).

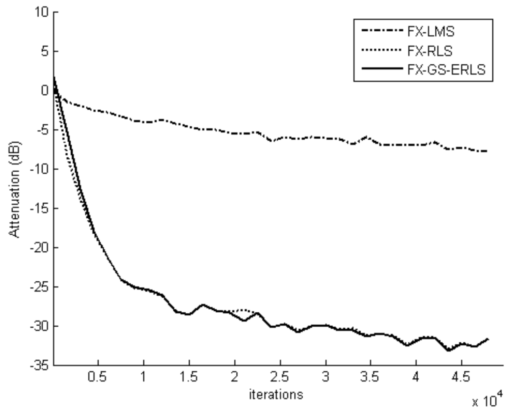

Fig. 7. Convergence of multichannel filtered-x algorithms for ANC, with ideal plant models ($I=1, J=3, K=2, L=128$, $M=64$).

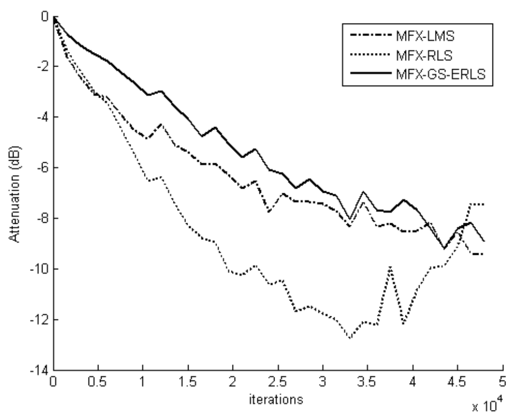

Fig. 8. Convergence curves of multichannel delay-compensated modified filtered-x algorithms for ANC, with 10 dB SNR plant models ($I=1, J=3, K=2, L=128$, $M=64$).

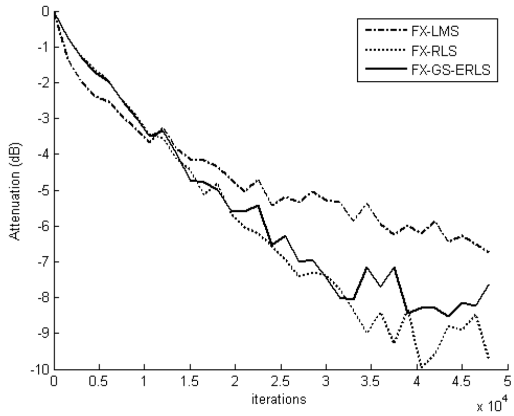

Fig. 9. Convergence curves of multichannel filtered-x algorithms for ANC, with 10 dB SNR plant models ($I=1, J=3, K=2, L=128$, $M=64$).

The value of η has to be reduced to 0.1 for the MFX-RLS, FX-RLS, MFX-GS-ERLS and FX-GS-ERLS algorithms in order to assure stability, at the price of reduced convergence speed. In this case, the behavior of the FX-GS-ERLS algorithm is better than that of the MFX-GS-ERLS algorithm. Therefore, the FX-GS-ERLS algorithm is potentially more robust to inaccuracies of the plant model. However, despite its increased numerical complexity, it does not bring important convergence gain over the MFX or FX versions of the LMS algorithm.

5 Conclusions

The FX-GS-ERLS algorithm based on the Gauss-Seidel method is introduced for practical active noise control systems using FIR adaptive filtering. This algorithm and its MFX version were compared with the previously published MFX-RLS and FX-RLS algorithms. It was shown that the proposed algorithm has a considerably reduced numerical complexity in comparison with their RLS counterparts. Also, it is proved FX-GS-ERLS and MFX-GS-ERLS algorithms have similar convergence properties for ideal and noisy plants models.

Acknowledgments

This work was supported by the UEFISCSU grant PN-II-PCE “Idei” no. 331/01.10.2007

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The codes for the proposed algorithms can be obtained from
http://falbu.50webs.com/List_of_publications_anc.htm

The reference for the paper is:

F. Albu, C. Paleologu, "New filtered-x recursive least square algorithms for active noise control", in *Proc. of INTERNOISE* 2010, Lisbon, June 2010, pp. 4540-4549