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New multichannel modified filtered-x algorithms for active noise control using the dichotomous coordinate descent method

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In this paper, several multichannel modified filtered-x algorithms for active noise control systems using the dichotomous coordinate descent method (DCD) are introduced. This multiplier-less and division-less method is used for avoiding the matrix inversion that appears in adaptive algorithms such us recursive least square (RLS) based algorithms, affine projection (AP) or its fast versions. The study is focused on the important computational savings given by the use of DCD method, the effect on the convergence properties and stability of the investigated algorithms. A comparison of their convergence performance in case of using non-ideal noisy acoustic plants is also given. It is proved by simulations that the use of the dichotomous coordinate descent method can be an interesting option for reducing the computational cost of practical multichannel algorithms for ANC systems.

1 Introduction

Active noise control (ANC) systems are being increasingly researched and developed [1]. The delay compensated or modified filtered-x structure for active noise control systems using FIR adaptive filtering was introduced in [2], and it is presented in Fig. 1. The structure in Fig. 1 eliminates the plant delay by computing an estimate of the primary field signals, which are unaffected by the changes of the adaptive FIR filter coefficients.

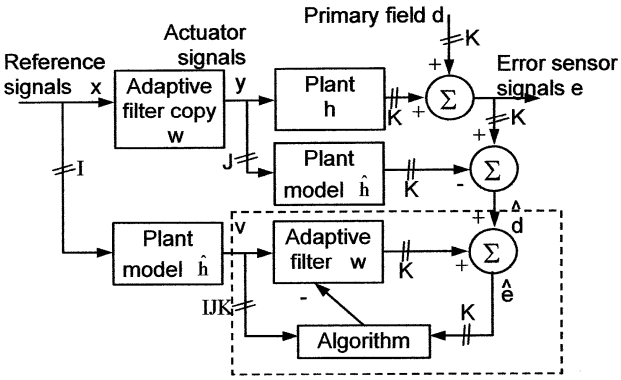

Fig.1 A delay compensated or modified filtered-x structure for active noise control.

For ANC using adaptive FIR filters, the multi-channel filtered-x least-mean-square (FX-LMS) algorithm [1], [2] is the most commonly used algorithm. The drawback of the FX-LMS algorithm is the slow convergence speed, especially for broadband multi-channel systems. Although it converges faster than the FX-LMS algorithm, the delay compensated or modified filtered-x LMS algorithm (MFX-LMS) [2], [3] also suffers from the same slow convergence problem, especially for multi-channel systems. For ideal (not noisy) plant models, FAP algorithms typically may not provide the same convergence speed as recursive least-squares (RLS) based algorithms [3], [4]. However, they demonstrate a much improved convergence speed compared to FX-LMS and MFX-LMS algorithms, without the high increase of the computational load or the numerical instability often found in RLS-based algorithms, especially for multichannel systems [3], [4].

It is well known that the stochastic gradient descent algorithms have poor convergence speed, while the recursive-least-squares algorithms are too complex and often numerically unstable (see, e.g. [3] and the references therein). An affine projection algorithm for multichannel active noise control called the Modified Filtered-X Affine Projection algorithm (MFX-AP) has been presented in [4]. This algorithm it is still too complex for practical applications. Therefore, simpler fast affine projection (FAP) algorithms suitable for active noise control and based on some approximations of the original affine projection algorithm has been proposed in [4], [5]. All these algorithms need at least one inverse matrix computation, which is very complex for large matrices and prone to numerical instability. In [6] the dichotomous coordinate descent (DCD) algorithm has been proposed and in [7] has been used for solving the implicit linear system of the MFX-AP equations. The resulting efficient algorithm was called the modified filtered-x dichotomous coordinate descent affine projection (MFX-DCDAP) algorithm and is presented in Section 2.1.

The RLS based algorithms have a faster convergence speed. A new algorithm called the multi-channel delay-compensated Modified Filtered-X Dichotomous Coordinate Descent Approximated Exponential Recursive Least Square (MFX-DCDAERLS) is obtained in Section 2.2. Section 3 presents simulation results of the DCD based algorithms and their original counterparts. A comparison of the numerical complexity of the proposed algorithm with the classical RLS algorithm is presented in Section 4. Section 5 concludes this work.

2 Dichotomous coordinate descent based algorithms for active noise control

The DCD algorithm is based on binary representation of elements of the solution vector with M_b bits within an amplitude range $[-H, H]$ [6]. Another parameter of the DCD algorithm is N_u, that represent the maximum number of "successful" iterations when a bit update happens. More details about the DCD algorithm can be found in [6] and [8]. If H is a power of two the DCD algorithm is implemented only with additions and bit shifts operations [6]. Thus, the DCD algorithm can be implemented without explicit multiplications and divisions. The peak complexity of the DCD algorithm for given M_b and N_u, is $N(2N_u + M_b)$ shift-accumulation (SACs) operations [6].

2.1 The modified filtered-x dichotomous coordinate descent affine projection algorithm

The notations used in this paper are taken from [7]. The variable n refers to the discrete time, I is the number of reference sensors, J represents the number of actuators, K - number of error sensors, L is the length of the adaptive FIR filters, M is the length of (fixed) FIR filters modeling the plant and N is the projection order. The vectors $\mathbf{x}_i = [x_i(n), \cdots, x_i(n-L+1)]^T$ and $\mathbf{x}_i^{'} = [x_i(n), \cdots, x_i(n-M+1)]^T$ consist of the last L and M samples of the reference signal $x_i(n)$, respectively. The vector $\mathbf{y}_j = [y_j(n), \cdots, y_j(n-M+1)]^T$ consists of the last M samples of the actuator signal $y_j(n)$. The samples of the filtered reference signal $v_{i,j,k}(n)$ are collected in the $IJ \times K$, and $IJL \times K$ matrices $\mathbf{V}_0(n) = \begin{bmatrix} v_{1,1,1}(n) & \cdots & v_{1,1,K}(n) \\ \vdots & \ddots & \vdots \\ v_{I,J,1}(n) & \cdots & v_{I,J,K}(n) \end{bmatrix}$, $\mathbf{V}(n) = \left[\mathbf{V}_0^T(n) \ \cdots \ \mathbf{V}_0^T(n-L+1)\right]^T$. The vectors $\hat{\mathbf{d}}(n) = \left[\hat{d}_1(n), \ \hat{d}_2(n), \ \cdots \ \hat{d}_K(n)\right]$ and $\hat{\mathbf{e}}(n) = \left[\hat{e}_1(n), \ \hat{e}_2(n), \ \cdots \ \hat{e}_K(n)\right]$ consist of estimates $\hat{d}_k(n)$ of the primary sound field $d_k(n)$ and of alternative error signals samples $\hat{e}_k(n)$, both computed in delay-compensated modified filtered-x structures. The vector $\mathbf{h}_{j,k} = \left[h_{j,k,1}, \cdots, h_{j,k,M}\right]^T$ consists of taps $h_{j,k,m}$ of the (fixed) FIR filter modeling the plant between signals $y_j(n)$ and $e_k(n)$. The $IJL \times 1$ vector $\mathbf{w}(n) = \left[\left[w_{1,1,1}(n) \ \ldots \ w_{I,J,1}(n)\right] \ \cdots \ \left[w_{1,1,L}(n) \ \cdots \ w_{I,J,L}(n)\right]\right]^T$ consists of the coefficients from all the adaptive FIR filters linking the signals $x_i(n)$ and $y_j(n)$. Finally, $e_k(n)$ is the k^{th} error sensor signal, μ is a normalized convergence gain $0 \le \mu \le 1$, $\mathbf{I}_{KN}$ is an identity matrix of size $KN \times KN$ and δ is a regularization factor that may be used to help with eventual numerical instability.

$$y_j(n) = \sum_{i=1}^{I} \mathbf{w}_{i,j}^T(n)\mathbf{x}_i(n) \tag{1}$$

$$v_{i,j,k}(n) = \mathbf{h}_{j,k}^T \mathbf{x}_i'(n) \tag{2}$$

$$\hat{d}_k(n) = e_k(n) - \sum_{j=1}^{J} \mathbf{h}_{j,k}^T \mathbf{y}_j(n) \tag{3}$$

$$\hat{\mathbf{e}}^T(n) = \hat{\mathbf{d}}^T(n) + \mathbf{V}^T(n)\mathbf{w}(n) \tag{4}$$

If we note by $\mathbf{R}_{KN}(n)$ the autocorrelation matrix (size $KN \times KN$) and by $\mathbf{P}(n)$ we have

$$\begin{aligned} \mathbf{P}(n) &= \left(\mathbf{V}^T(n)\mathbf{V}(n) + \delta \mathbf{I}_{KN}\right)^{-1} \hat{\mathrm{e}}_N^{\ T}(n) = \\ &= \left(\mathbf{R}_{KN}(n) + \delta \mathbf{I}_{KN}\right)^{-1} \hat{\mathbf{e}}_N^T(n) \end{aligned} \tag{5}$$

The weights are obtained by the following equation:

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu \mathbf{V}(n)\mathbf{P}(n) \tag{6}$$

Therefore, the MFX-DCDAP is defined by the equations (1)-(6) [7]. Also, an example of a DCD based fast version of the pseudo affine projection algorithm, suitable for ANC systems can be found in [8].

2.2 The modified filtered-x dichotomous coordinate descent approximated exponential recursive least square algorithm

In order to use the DCD algorithm in a least square algorithm, we use the formulation of the RLS problem in terms of a sequence of auxiliary normal equations with respect to increments of the filter weights. This approach was applied to the exponentially weighted case and a new structure of the RLS algorithm was derived [9]. An approximation in solving the auxiliary linear system of the transversal Exponential RLS (ERLS) algorithm is used. This algorithm is adapted to the multichannel case. To describe the multi-channel delay-compensated Modified Filtered-X Approximated Exponential Recursive Least Square algorithm more notations have to be defined:

$\mathbf{R}(n)$ (size $IJL \times IJL$) is initialized as an identity matrix multiplied by the regularization factor δ. We have

$$\mathbf{R}(n) = \lambda \mathbf{R}(n-1) + \mathbf{V}(n)\mathbf{V}^T(n) \tag{7}$$

,where λ is a forgetting factor ($0 < \lambda < 1$)

$\overline{\mathbf{w}}(n)$ is a $IJL \times 1$ vector used in solving the auxiliary equations [9]. If we assume that the residual vector described in [9] is null each iteration, the DCD algorithm can be used to solve the following linear system:

$$\mathbf{R}(n)\overline{\mathbf{w}}(n) = \mathbf{V}(n)\hat{\mathbf{e}}^T(n) \tag{8}$$

The weights are given by the following equation

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \overline{\mathbf{w}}(n) \tag{9}$$

Therefore, the MFX-DCDAERLS is defined by the equations (1)-(4) and (7)-(9). Also, other DCD based algorithms could be derived from known fast RLS algorithms (see, e.g. [3] and the references therein) and adapted for ANC systems. The DCD algorithm can replace some implicit linear systems encountered in fast RLS algorithm's equations.

3 Simulations

The DCD based algorithms (MFX-DCDAERLS and MFX-DCDAP) were simulated and compared to the previously published multi-channel modified filtered-x affine projection algorithm (MFX-AP, [3]) and the multi-channel modified filtered-x RLS algorithm (MFX-RLS, [3]). We used in our simulation $\lambda = 0.995$ and the reference signal was a white noise with zero mean and variance one. The simulations were performed with acoustic transfer functions experimentally measured in a duct. The impulse responses used for the multi-channel acoustic plant had 64 samples each ($M = 64$), while the adaptive filters had 100 coefficients each ($L = 100$). The step size μ for the AP algorithms was 1. The performance of the algorithms was measured by

$$Attenuation(dB) = 10 \cdot \log_{10} \frac{\sum_k E\left[e_k^2(n)\right]}{\sum_k E\left[d_k^2(n)\right]} \quad (10)$$

Fig. 2 compares the performance of the selected algorithms with ideal plant models, for a multi-channel ANC system obtained from Matlab™ implementations of the algorithms ($I = 1$, $J = 3$, $K = 2$ were used). For the AP algorithms a projection order of $N = 5$ was used. Fig. 2 shows that the MFX-DCDAP algorithm obtains almost identical performance with MFX-AP for $N_u = 8$ iterations and $M_b = 16$ bits. Also, it can be seen that the MFX-DCDAERLS and MFX-RLS algorithms have the same initial convergence speed. However, the MFX-DCDAERLS algorithm has a loss in attenuation due to the approximation used in deriving the algorithm. Simulations have shown that this approximation increased the numerical robustness of the algorithm. As expected, the MFX-RLS and MFX-DCDAERLS convergence performance is better than that of the MFX-AP algorithm.

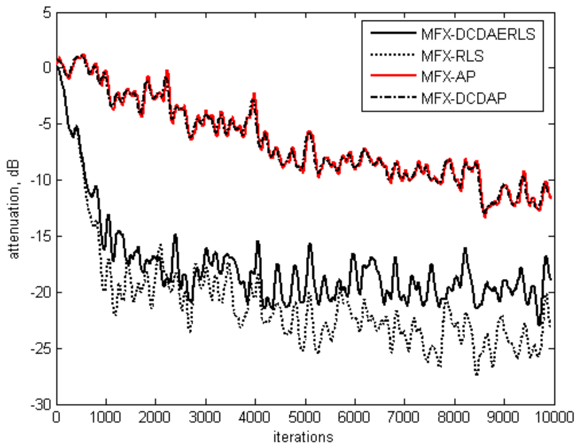

Fig.2 Convergence curves of multichannel delay-compensated modified filtered-x algorithms for ANC, with ideal plant models ($I = 1$, $J = 3$, $K = 2$; $N_u = 8$, $M_b = 16$, and $H = 1/2$ were used for DCD versions)

Fig. 3 shows the norm difference between the ideal and DCD computed linear system solutions for different N_u and M_b values. As expected the error decreases for higher N_u and M_b values. However, the error difference is smaller after the algorithm converges. This suggests that the N_u and M_b parameters can be reduced when the algorithm converges. The implementation using 8 DCD iterations and 16 bits provides a very small dB difference between the DCD based AP version and the AP algorithm that uses the ideal matrix inverse. The average DCD complexity is around 60% of the theoretical SAC peak complexity. The DCD part increases the number of additions, but it has no divisions or multiplications.

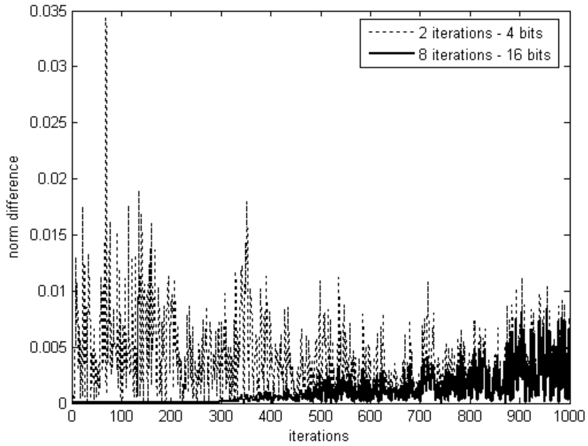

Fig.3. The norm difference between ideal and DCD computed system solutions for different N_u and M_b values

So far ideal plant models had been assumed. The noise added to the ideal plant models was added on a frequency by frequency basis, where a random complex value with a magnitude of 20 dB less that the original magnitude was added to each frequency in the frequency response [3]. Fig. 4 shows the performance when plant models with a 10 dB SNR were used in the monochannel case. It can be seen that the MFX-RLS algorithm performance is the most affected, and its known potential numerical instability is exacerbated. The other considered algorithms are more robust to noisy plant models. Therefore, the proposed DCD based algorithms are interesting options for practical implementations.

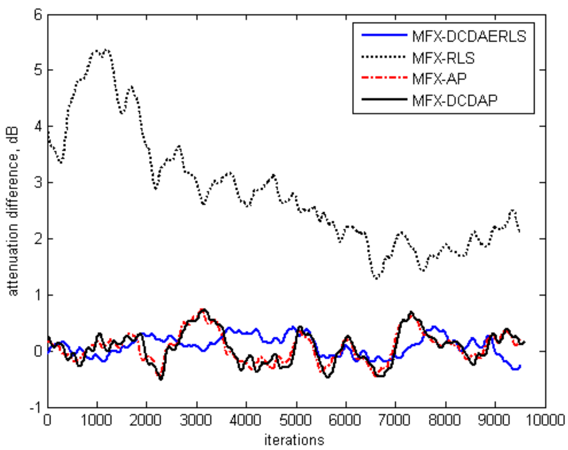

Fig. 4. a) Attenuation difference between the multichannel delay-compensated modified filtered-x ANC algorithms with ideal plant models and those using plant models with a SNR of 20 dB ($I = 1$, $J = 1$, $K = 1$, $N_u = 8$, $M_b = 16$, and $H = 1/2$);

4 Computational complexity

Matrix inversions of the initial algorithms were assumed to be performed with standard LU decomposition: $O\{X^3/2\}$ multiplies, where X is the size of a square matrix. The numerical complexities of the considered algorithms is measured by the number of multiplications per algorithm iteration [3]:

$$C_{MFX-DCDAP} = IJK(M + 2L + 2KN) + IJL + JKM \quad (11)$$

$$C_{MFX-AP} = IJL(2 + 2KN + K^2N^2) + JKM(1 + I) + \\ + K^2N^2 + K^3N^3/2 \quad (12)$$

$$C_{MFX-RLS} = L^2\left(2I^2J^2K + I^2J^2\right) + L\left(2IJK^2 + 3IJK + IJ\right) + \\ M\left(IJK + JK\right) + K^2 + K + K^3/2 \quad (13)$$

$$C_{MFX-DCDAERLS} = IJL\left(IJL\left(K+1\right)/2 + 2K + 1\right) + JKM\left(I+1\right) \quad (14)$$

Algorithm for multichannel ANC, L=100, M=64, N=5	Multiplies per iteration for I=1, J=1, K=1	Multiplies per iteration for I=1, J=3, K=2
MFX-DCDAP	**438**	**2388**
MFX-AP	3916	37968
MFX-DCDAERLS	**10428**	**137268**
MFX-RLS	30730	455278

Table 1. Comparison of computational load of the DCD based algorithms with the original delay-compensated modified filtered-x algorithms for ANC

It can be seen from Table 1 that the DCD based MFX algorithms are much less complex than their counterparts. The reduction in complexity depends on the values of the parameters (*I, J, K, L, M)*. However, for the mono-channel case the reduction is at least 66%, while it is even about 93 % for the AP algorithm in case of the multi-channel case.

5 Conclusion

The use of the dichotomous coordinate descent algorithm has been investigated in order to avoid matrix inversions encountered in typical ANC algorithms. It is shown that important computational savings can be obtained. Also, the convergence properties of the proposed algorithms with ideal and noisy acoustic plants have been studied. It is proved by simulations that the proposed DCD based algorithms can be an interesting option for practical ANC systems.

Acknowledgments

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