TITLE: THE GAUSS-SEIDEL PSEUDO AFFINE PROJECTION ALGORITHM AND ITS APPLICATION FOR ECHO CANCELLATION

ABSTRACT

In this paper we propose a new approach for adaptive echo cancellation: the Gauss-Seidel Pseudo Affine Projection algorithm (GSPAP). It is proved by simulations that it is stable, fast convergent and has good tracking abilities. The computational complexity of the GSPAP algorithm is evaluated. It is shown that its simplified and division-less version achieves much improved performances and is only marginally more complex than the NLMS algorithm.

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INTRODUCTION

In echo cancellation systems an adaptive filter algorithm is used to reduce the echo. The well-known normalized LMS (NLMS) algorithm has been widely used but it has slow asymptotic convergence. The affine projection algorithm (APA) performance rivals the recursive-least square algorithms in many situations, but its complexity is $2LN + 7N^2$ where L is the length of the filter and N is the projection order [1]. Its fast version proposed in [2], when implemented with an embedded FRLS (Fast Recursive Least Squares) algorithm suffers from numerical instability. In [3] we proposed a simpler and stable FAP algorithm whose complexity is $2L + N^2 + 4N + 1$ multiplies and divisions. In this paper we'll derive a new stable and simpler FAP algorithm based on the Gauss-Seidel method called the Gauss-Seidel Pseudo Affine Projection Algorithm (GSPAP) and examine its application for digital echo cancellers and acoustic echo cancellation systems.
Assuming that the input signal is stationary the following relation is obtained [4]

$$\mathbf{R}_{\mathbf{t},N}\left(1\, a_1 \ldots a_{N-1}\right) = \left(\mathbf{X}_{\mathbf{t}}^T \mathbf{U}_{\mathbf{t}}, 0, \ldots 0\right) \quad (1)$$

where $\left[1\, a_1 \,..\, a_{N-1}\right]^T$ represent the optimal forward linear prediction coefficient vector, $\mathbf{R}_{\mathbf{t},N}$ is the auto-correlation matrix of the input signal, $\mathbf{X}_{\mathbf{t}} = \left[x_t, \ldots, x_{t-L+1}\right]^T$ and $\mathbf{U}_{\mathbf{t}} = \mathbf{X}_{\mathbf{t}} + \sum_{i=1}^{N-1} a_i \mathbf{X}_{\mathbf{t}-i}$, x_t is the input signal and y_t is the desired output signal [4]. The relation (1) can be re-written as $\mathbf{R}_{t,N}\, \underline{P} = \underline{b}$, if we note $\underline{P} = \left[1/\mathbf{X}_{\mathbf{t}}^T\mathbf{U}_{\mathbf{t}}, a_1/\mathbf{X}_{\mathbf{t}}^T\mathbf{U}_{\mathbf{t}}, \ldots, a_{N-1}/\mathbf{X}_{\mathbf{t}}^T\mathbf{U}_{\mathbf{t}} \right]^T$ and $\underline{b}$ is an N vector with only one non-zero element, which is unity at the top. We solve this linear system using one Gauss-Seidel iteration. We have $\mathbf{U}_{\mathbf{t}} = \frac{1}{\underline{P}_0} \sum_{i=0}^{N-1} \underline{P}_i \mathbf{X}_{\mathbf{t}-i}$ and the coefficient update equation is given by

$$\mathbf{H}_t = \mathbf{H}_{t-1} + \frac{\mu}{\mathbf{U}_t^T \mathbf{X}_{\mathbf{t}} + \delta} \mathbf{U}_{\mathbf{t}} \left(y_t - \mathbf{X}_t^T \mathbf{H}_{\mathbf{t-1}} \right) \quad (2)$$

where δ is a regularization factor and μ is the step size. An advantage of this algorithm is that it provides the filter coefficients unlike the other FAP algorithms that compute only auxiliary coefficients. Therefore, it can be applied for other voice applications.

The Gauss-Seidel Pseudo Affine Projection algorithm has $2L + N^2 + 3N + 5$ multiplies and divisions. However further processing cycles savings are possible. We found that a projection value of 4 is enough for the unvoiced sections while it should be higher for voiced section. Imposing a simple threshold to $r_0(t)$ of the autocorrelation estimation at lag 0 lead to the choice of the projection order for the chosen frame. Also, this variable has a positive value that slowly changes over time (see Figure 4). Our approach in eliminating the division is similar to the approximation involved in adaptive delta modulation systems. The division can be replaced by the following expression:

$$\hat{r}_0(t) = \hat{r}_0(t-1) + 2^{-sign(\partial(t))} \cdot \alpha \cdot \partial(t) \qquad (3)$$

where $\hat{r}_0(t)$ is an estimation of $1/r_0(t)$ and $\partial(t) = 1 - \hat{r}_0(t-1) \cdot r_0(t)$. A power of two is indicated for α in order to replace multiplication by shifting operations.

The GSPAP and NLMS algorithms were firstly used for a system identification example. The input signal is a composite source signal according to the G.168 ITU recommendation for digital echo cancellers. No noise was added and the echo path is taken from that standard. Figure 1 shows clearly the superiority of the GSPAP algorithm. Therefore this algorithm can be used for digital echo cancellers. In our acoustic echo cancellation simulations the excitation signal is a speech signal, sampled at 8 kHz. Figures 2 and 3 confirm the faster convergence and better tracking performance of the GSPAP algorithm to the NLMS for AEC applications. Figure 4 shows that using a variable affine projection order don't affect the overall performance of the GSPAP algorithms and it could even lead to some small improvements for some speech frames. In our simulated example the savings are about 64 % for the Gauss-Seidel section. Figure 5 illustrates that the iterated method approximates well the exact inverse. Depending on the DSP processor the real time savings of the approximated division is about 70 % compared with the division operation. Table 1 compares the numerical complexity of the considered algorithms for two values of the projection order. It can be seen that GSPAP is only marginally more complex than NLMS. This algorithm obtains the performance of high order affine projection algorithm at much less computational effort than the second order exact affine projection algorithm (see Table 1). The values between accolades from Table 1, column of $N = 10$ indicates the average number of multiplies and divisions when the update of $\underline{P}$ vector is made every 10 samples. We investigated the behaviour of the algorithm by simulating it using 16-bit fixed point and 32-bit floating point. Figure 6 shows some losses in performances due to lower finite precision. The GSPAP finite precision implementation was stable in all of our simulations.

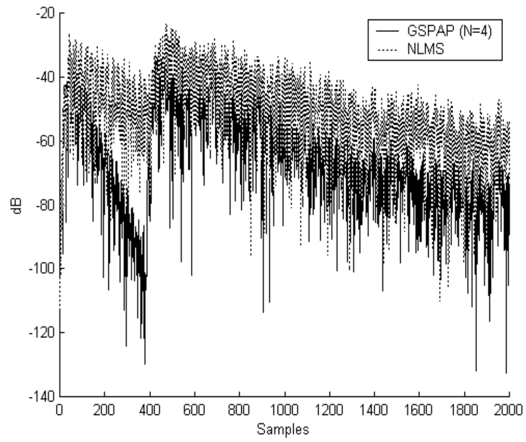

Figure 1. The square errors for the NLMS and GSPAP algorithm (*L*=128, *N*=4, *p*=10)

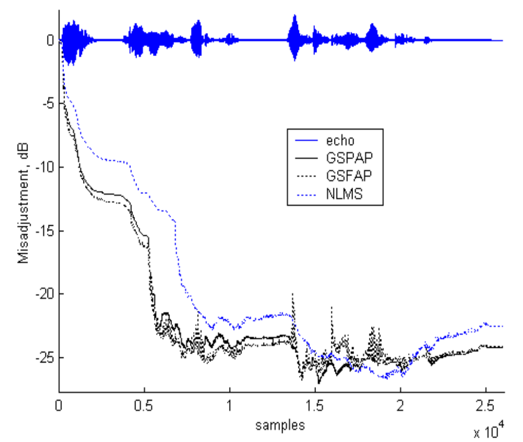

Figure 2. The learning curves for GSFAP, GSPAP and NLMS algorithms (L=256, N=8)

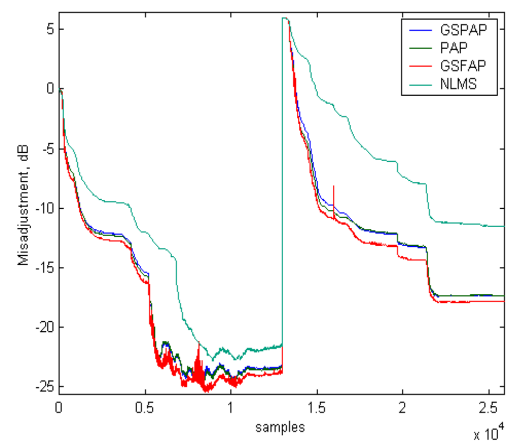

Figure 3. The learning curves for GSFAP, PAP, GSPAP and NLMS algorithms (PAP and GSPAP curves are almost co-incidental most of the time, L=256, N=8) under a sudden change in the echo path

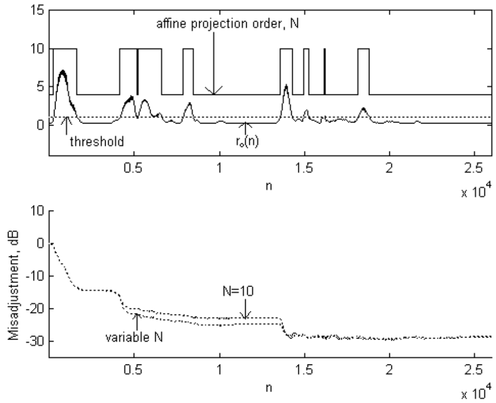

Figure 4. The autocorrelation at lag 0, the variable affine projection order obtained by using a threshold (top) and the learning curves of the GSPAP algorithm for a fixed and variable affine projection order (bottom).

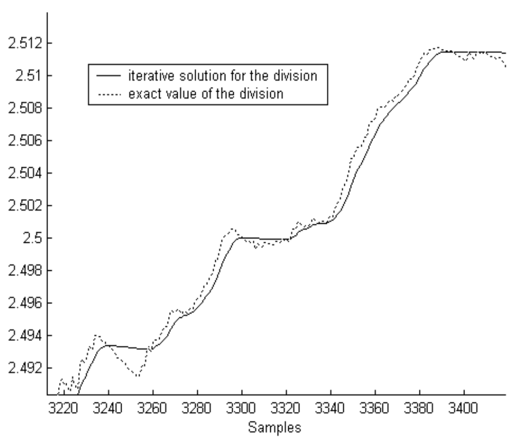

Figure 5. Zoom of the exact result of the inverse (full line) and the estimated iterative result (dotted line)

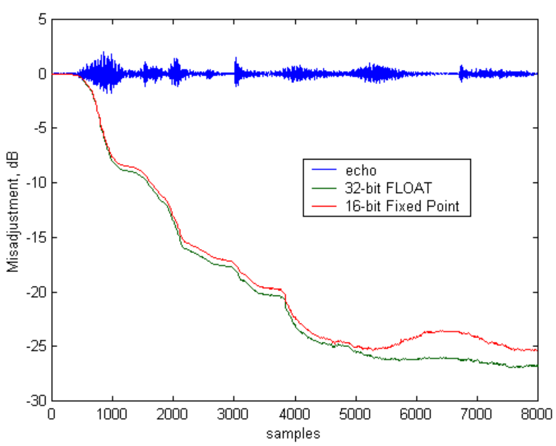

Figure 6. The learning curves for 32-bit FLOAT and 16-bit fixed point GSPAP implementations (L=500, N=10, p=4)

Table 1. Computational complexity (number of divisions and multiplies) of different FAP algorithms (L=1000)

ALGORITHM	DETAILED	$N=2$	$N=10$
APA [1]	$2LN+7N^2$	4028	20700
FAP [2]	$2L+20N$	2040	2200
GSFAP[3]	$2L+N^2+4N+1$	2013	2141 (2051)
PAP[4]	$2L+N^2+5N+4$	2018	2154 (2064)
GSPAP	$2L+N^2+3N+5$	2015	2135 (2045)
NLMS	$2L+4$	2004	2004

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The matlab code for the GSFAP and GSPAP algorithms can be obtained by filing the form from http://falbu.50webs.com/gs.html

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