A Fast Filtering Block-Sparse Proportionate Affine Projection Sign Algorithm

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***Abstract*—In this paper, a new proportionate affine projection sign algorithm for block-sparse identification using a fast recursive filtering procedure and dichotomous coordinate descent iterations is proposed. It is shown that the proposed algorithm is a good candidate for network echo cancellation systems operating under impulsive environments.**

***Keywords*—Proportionate-type algorithms; adaptive filters; affine projection sign algorithm; fast recursive filtering;**

I. Introduction

The adaptive filters are used in many applications such us network echo cancellation [1], acoustic echo cancellation [2], and active noise control [3]-[4] etc. In network echo cancellation application, the echo paths are usually long and sparse [1]. The normalized least mean square (NLMS) or affine projection algorithm (APA) families were used for many system identification applications [5]. The sparseness of the echo path is better used by the algorithms that adjust the filter coefficients in proportion with their magnitude. Such examples are proportionate NLMS (PNLMS) [6] and its improved PNLMS (IPNLMS) [7]. The memory improved PAPA (MIPAPA) [8] and its approximate version [9] were also proposed. In [10] a numerical complexity reduction is achieved by employing a fast recursive filtering procedure.

The affine projection sign algorithm (APSA) [11] and its efficient implementation [12] were proposed for dealing with impulsive noise interference, a known drawback of above mentioned algorithms. Superior robustness to such conditions has been obtained by the real-coefficient improved proportionate affine projection sign algorithm (RIP-APSA) [13] and memory improved proportionate APSA (MIP-APSA) [14]. Recently, a block-sparse family of algorithms were introduced for the proportionate type of algorithms [15]-[16]. The block sparse MIP-APSA (BS-MIP-APSA) was proposed for block-sparse identification under impulsive noise [17]. The sparsity can also be exploited by using the norm-adaption penalized least mean square/fourth algorithm [18].

In this paper, the fast recursive filtering procedure from [10], [12] and the block-sparse approach from [15]-[17] are adapted to the RIP-APSA. The proposed algorithm is called block-sparse fast recursive RIP-APSA (BS-FRRIP-APSA).

The paper is organized as follows. Section II introduces the proposed algorithm and its equations. Simulation results and future work are presented in Section III while Section IV concludes this work.

II. The Proposed Algorithm

In an echo cancellation application an adaptive filter that models the L-length echo path, $\mathbf{h}$, is defined by $\mathbf{w}(k)=\left[w_0(k),w_1(k),...,w_{L-1}(k)\right]^T$, where k represents the time index. We note by $x(k)$ the far-end signal, $z(k)$ the near-end signal and $v(k)$ the background noise signal. The desired signal is $y(k)=\mathbf{x}^T(k)\mathbf{h}+z(k)+v(k)$ where $\mathbf{x}(k)=\left[x(k),...,x(k-L+1)\right]^T$. The output of the adaptive filter is $\hat{y}(k)=\mathbf{x}^T(k)\mathbf{w}(k)$, and the error vector is given as

$$\mathbf{e}(k)=\mathbf{y}(k)-\mathbf{X}^T(k)\mathbf{w}(k), \tag{1}$$

where $\mathbf{y}(k)=\left[y(k),y(k-1),...,y(k-M+1)\right]^T$, is a $M\times 1$ vector, M is the projection order, and $\mathbf{X}(k)=\left[\mathbf{x}(k),\mathbf{x}(k-1),...,\mathbf{x}(k-M+1)\right]$, is the input signal matrix. The proportionate matrix is $\mathbf{G}(k)=\mathrm{diag}\left\{g_0(k),...,g_{L-1}(k)\right\}$, where the proportionate factors, $g_l(k)$ are the following:

$$g_l(k)=\frac{1-\alpha}{2L}+\frac{|w_l(k)|(1+\alpha)}{2\sum_{i=0}^{L-1}|w_i(k)|+\varepsilon},\ l=1..L, \tag{2}$$

where ε is a small positive constant and the absolute value of α is smaller or equal to 1 [14].

If we note by $\mathbf{x}_{gs}(k)=\mathbf{G}(k)\mathbf{X}(k)\mathrm{sgn}\left(\mathbf{e}(k)\right)$, where $\mathrm{sgn}(\cdot)$ is the sign function, and the RIP-APSA's weight updating equation is the following [14]

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$$\mathbf{w}(k+1)=\mathbf{w}(k)+\frac{\mu\mathbf{x}_{gs}(k)}{\sqrt{\delta+\mathbf{x}_{gs}^{T}(k)\mathbf{x}_{gs}(k)}}, \qquad (3)$$

where μ is the step size and δ is a small constant.

In [15], the impulse response is partitioned in blocks of length S and the proportionate idea is applied on blocks. The block-sparse scheme for BS-PAPA was derived in [16] and based on the optimization of the $l_{2,1}$ norm. The $l_{2,1}$ norm was defined as [17]

$$\|\hat{\mathbf{w}}\|_{2,1}=\sum_{i=1}^{N}\sqrt{\hat{\mathbf{w}}_{[i]}^{T}\hat{\mathbf{w}}_{[i]}}, \qquad (4)$$

where

$$\hat{\mathbf{w}}_{[i]}=\left[\hat{w}_{(i-1)S+1},\hat{w}_{(i-1)S+2},\ldots,\hat{w}_{iS}\right]^{T}, \qquad (5)$$

S being a group size parameter and $N = L/S$ [17].

The proportionate matrix is computed as follows

$$\mathbf{G}(k-1)=diag\left[g_1(k-1)\mathbf{1}_S,\ldots,g_N(k-1)\mathbf{1}_S\right]. \qquad (6)$$

In (6) $\mathbf{1}_S$ is the S-length row vector of all ones and the proportionate coefficients are computed as follows

$$g_l(k-1)=\frac{1-\alpha}{2L}+\frac{(1+\alpha)\left\|\hat{\mathbf{w}}_l(k-1)\right\|_2}{2\sum_{i=0}^{N-1}\left\|\hat{\mathbf{w}}_i(k-1)\right\|_2+\varepsilon}. \qquad (7)$$

In [12] a fast filtering procedure has been adapted to APSA. Using the same idea and the proportionate matrix of (6) for RIP-APSA, the block-sparse fast recursive RIP-APSA (BS-FRRIP-APSA) is obtained.

We note by $\mathbf{P}(k)=\mathbf{G}(k)\mathbf{X}(k)$, $\mathbf{R}(k)=\mathbf{P}^T(k)\mathbf{P}(k)$, $\hat{\mathbf{y}}(k)=\mathbf{X}^T(k)\mathbf{w}(k)$, $\mathbf{H}(k)=\mathbf{X}^T(k)\mathbf{P}(k-1)$, and $\mathbf{z}(k)=\mathbf{X}^T(k)\mathbf{w}(k-1)$. Following the same steps from [10] the fast recursive implementation of BS-FRRIP-APSA is obtained and its equations are shown in Table 1.

The BS-FRRIP-APSA is slightly more complex than RIP-APSA. A source of this increase in complexity is given by the different computation of the proportionate matrix. All the computational saving tricks exposed in [10] and the references therein are inherited.

III. Simulation Results

The performance of BS-MIP-APSA [16], BS-FRRIP-APSA, and MRIP-APSA [10] were compared in terms of convergence speed and tracking abilities using the same simulated signals and conditions from [10] in the simulations. An AR(1) signal with a pole at 0.9 was used as the input signal. The SNR was 40 dB and the signal-to-interference ratio SIR was 0 dB modeled by a Bernoulli-Gaussian (BG) signal. The Bernoulli-Gaussian signal was obtained as a product of a Bernoulli process with the parameter Pr=0.002 and a Gaussian process, having a constant average power. We used $\delta=0.01$ and $\varepsilon=10^{-8}$.

The simulations were performed using three network impulse responses (NIR) with $L=1024$ coefficients. The echo path changed from the one-cluster sparse path (Fig. 1a) to the two-cluster sparse path (Fig. 1b) after 20000 samples and then to a dispersive NIR (Fig. 1c) after another 20000 samples.

Table I
The BS-FRRIP-APSA

Step	Initialization $\mathbf{e}(k)=0,\mathbf{w}(k)=0,\mathbf{x}(k)=0,\hat{\mathbf{y}}(k)=0,\sigma(k)=\varepsilon$ for $k\le 0$
	For $k=1,2,\ldots$
1	$\mathbf{z}(k)=\left[\mathbf{x}^T(k)\mathbf{w}(k-1),\ y^0(k-1),\ \ldots\ y^{M-2}(k-1)\right]^T$
2	Compute $\mathbf{G}(k-1)$ using (7) and $\mathbf{P}(k-1)$
3	$\mathbf{H}(k)=\mathbf{X}^T(k)\mathbf{P}(k-1)$
4	$\mathbf{q}(k)=\mathbf{H}(k)\operatorname{sgn}(\mathbf{e}(k-1))$
5	$\mathbf{s}(k)=\mu\sigma^{-1/2}(k-1)\mathbf{q}(k)$
6	$\hat{\mathbf{y}}(k)=\mathbf{z}(k)+\mathbf{s}(k)$
7	$\mathbf{e}(k)=\mathbf{y}(k)-\hat{\mathbf{y}}(k)$
8	$\mathbf{x}_{gs}(k)=\mathbf{P}(k)\operatorname{sgn}(\mathbf{e}(k))$
9	$\mathbf{R}(k)=\mathbf{P}^T(k)\mathbf{P}(k)$
10	$\sigma(k)=\sqrt{\delta+\operatorname{sgn}(\mathbf{e}^T(k))\mathbf{R}(k)\operatorname{sgn}(\mathbf{e}(k))}$
11	$\mathbf{w}(k+1)=\mathbf{w}(k)+\mu\sigma^{-1/2}(k)\mathbf{x}_{gs}(k)$

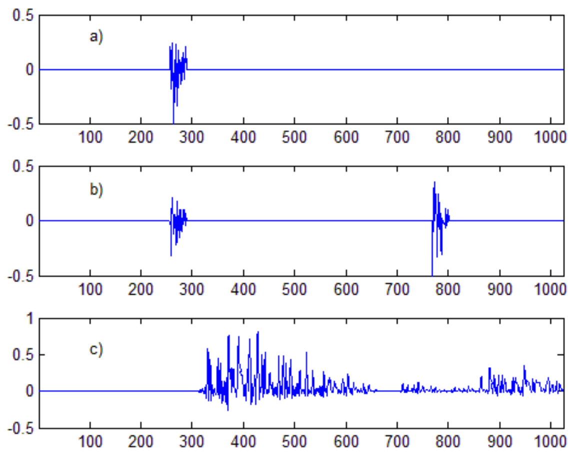

Fig. 1. Impulse responses: a) one-cluster block-sparse echo path, (b) two-cluster block-sparse echo path; c) dispersive echo path.

The normalized misalignment computed as $\eta(k) = 20 \cdot \log_{10}\left(\left\|\mathbf{h} - \mathbf{w}(k)\right\|_2 / \left\|\mathbf{h}\right\|_2\right)$ and averaged on 10 trials was plotted in order to compare the algorithms.

For the first experiment, the parameters of BS-FRRIP-APSA were $\mu = 0.01$, $\alpha = 0.5$ and $S = 16$. The projection order was varied and four M values were used: 1, 2, 4, and 8. Figure 2 shows that the convergence speed increases when the projection order increases while the steady-state misalignment increases.

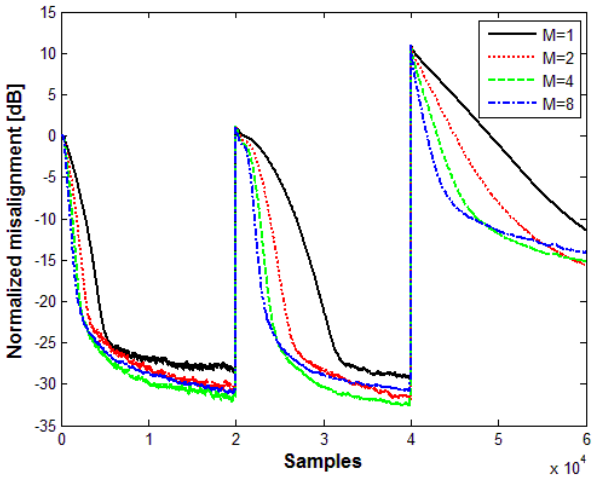

Fig. 2. Misalignment of BS-FRRIP-APSA with colored input signal, $\alpha = 0.5$ and $\mu = 0.01$ and different M values

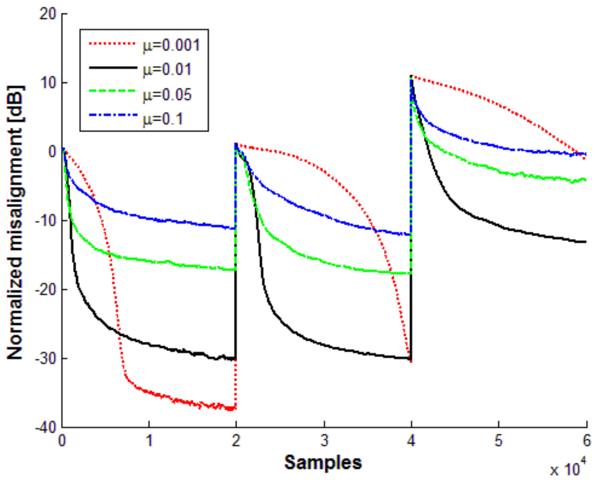

Fig. 3. Misalignment of BS-FRRIP-APSA with colored input signal, $M = 8$, $\alpha = 0.5$, and different μ values.

For the second experiment, the convergence and tracking performance of the BS-FRRIP-APSA using $\alpha = 0.5$, $M = 8$ and four μ values were used: 0.001, 0.01, 0.05, and 0.1. The same behavior as observed in Fig. 2 can be seen in Fig. 3 regarding the step size. This conclusion is valid for all the considered echo paths and the best results are obtained for $\mu = 0.01$.

In Fig. 4, the performance of the proposed BS-FRRIP-APSA for different group sizes S chosen as 1 (corresponds to IPAPSA), 16, 32, 128 and 1024 (corresponds to APSA) is investigated. The impact of different group sizes on BS-FRRIP-APSA is similar with that for BS-PAPA. The best performance speed for sparse paths is obtained for $S = 16$.

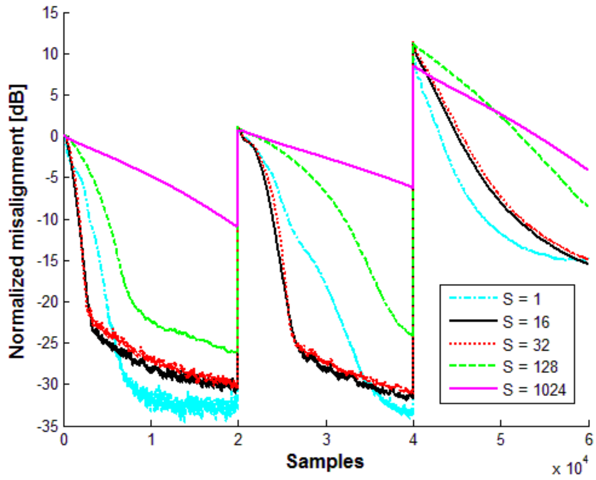

Fig. 4. Misalignment of BS-FRRIP-APSA for different group sizes, $M = 2$, colored input and SNR = 40 dB

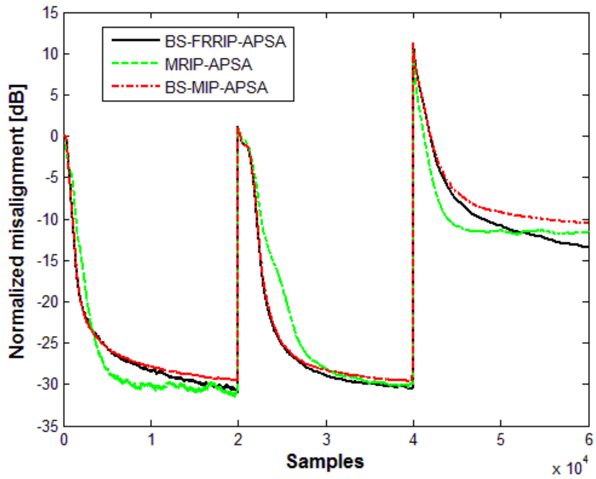

Fig. 5. Misalignment of BS-FRRIP-APSA, MRIP-APSA and BS-MIP-APSA with colored input signal, $M = 8$, $S = 16$, $\alpha = 0.5$ and $\mu = 0.01$.

The convergence and tracking performance of the MRIP-APSA [11], BS-MIP-APSA [17] and BS-FRRIP-APSA using $\mu = 0.01$, $\alpha = 0.5$, $M = 8$ and $S = 16$ are investigated in Fig. 5. It can be noticed from Fig. 5 that while the performance of BS-FRRIP-APSA is close to that of BS-MIP-APSA for one cluster and two cluster echo paths, it is better for the dispersive echo path. Also, it can be seen that MRIP-APSA has different convergence characteristics than the block sparse algorithms for all the investigated echo paths.

In Fig. 6 a speech input and the same parameters as above were used except M =2. It can be seen that BS-FFRIP-APSA and BS-MIP-APSA have a similar behavior for two-cluster and dispersive echo path. For the one-cluster echo path, BS-MIP-APSA achieves a lower misalignment value than BS-FFRIP-APSA. Also, it can be noticed from Figs. 2-6 that the performance of the investigated algorithms degrades with the decrease of the sparseness of the echo path.

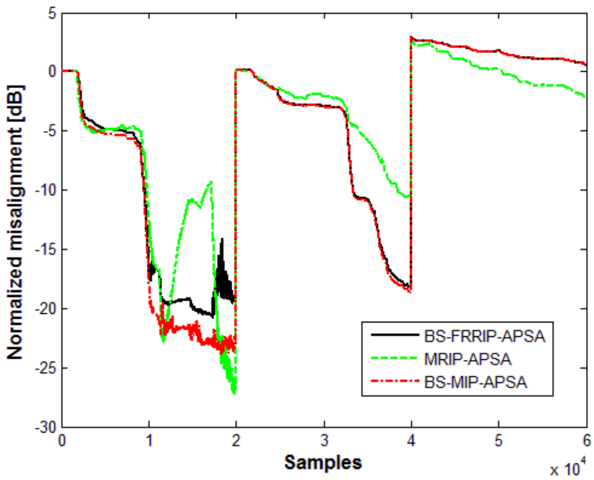

Fig. 6. Misalignment of BS-FRRIP-APSA, MRIP-APSA and BS-MIP-APSA with speech signal, $M = 2$, $S = 16, \alpha = 0.5$ and $\mu = 0.01$.

In [19] the proportionate algorithms showed good performance in an adaptive feedback cancellation application. Therefore, the potential of the proposed block-sparse algorithm for a two microphone approach for hearing aid devices will be investigated in the future work.

IV. Conclusions

In this paper, a fast recursive procedure together with the block-sparse approach were used in order to derive a new algorithm called BS-FFRIP-APSA. Simulations in the context of network echo cancellation for one-cluster, two-cluster and dispersive echo paths have shown that BS-FRRIP-APSA can be an alternative to BS-MIP-APSA and MRIP-APSA. The effect of the projection order and step-size of BS-FRRIP-APSA has been investigated as well.

Acknowledgment

The work of Felix Albu was supported by the CNCS-UEFISCDI PN-II-ID-PCE-2011-3-0097 grant of the Romanian National Authority for Scientific Research. The work of Jianming Liu and Steven L. Grant was supported by Wilkens Missouri Endowment.

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