

Complex Domain Adaptive System Identification Using Sparse Affine Projection Normalized Correlation Algorithms Under Impulsive Noises

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Abstract—Sparse adaptive filters are used extensively for enhancing the filter performance in a sparse system. The affine projection algorithm (APA) is effective in improving the convergence speed for strongly correlated input signals, but it is very sensitive to impulsive noise. Normalized Correlation Algorithm (NCA) is robust in impulsive noise environments. The affine projection normalized correlation algorithm (AP-NCA) used in complex-domain adaptive filters, combines the benefits of APA and NCA and it does not take into account the underlying sparsity information of the system. In this paper, we develop sparse AP-NCA algorithms to exploit system sparsity as well as to mitigate impulsive noise with correlated complex-valued input. Simulation results show that the proposed algorithms exhibit better performance than the AP-NCA for a sparse system. The robustness of these algorithms is evaluated in terms of Mean square error (MSE) performance in the adaptive system identification context.

Keywords—complex-domain adaptive filters; affine projection normalized correlation algorithm; impulsive noise; adaptive system identification

I. INTRODUCTION

In many real-life systems, the impulse response of the system is assumed to be sparse, containing only a few active taps in the presence of a large number of inactive taps [1-3]. Sparse systems are usually encountered in applications such as network and acoustic echo cancellers [4], wireless multipath channels [5, 6].

Adaptive filtering algorithms have received much attention over the past decades and are widely used for diverse applications such as system identification, interference cancellation, and channel estimation. In recent years, sparse adaptive filters have been developed to exploit the system sparse information and the performance can be greatly improved when compared with the conventional algorithms such as Least Mean Square (LMS) and Affine Projection Algorithm (APA) [7, 8]. Based on the assumption of the Gaussian noise model, sparse algorithms are derived by applying the ℓ_1 -norm relaxation into the LMS cost function [9, 10], sparsity-aware ℓ_p -norm penalized and reweighted ℓ_1 -norm penalized LMS algorithms are derived in [11, 12], and sparsity-aware affine projection adaptive algorithms for

system identification are proposed in [13-15]. However, these methods may be unreliable in estimating the systems under non-Gaussian impulsive noise environments. For example, the least mean square (LMS) [9] algorithm performance is affected by strong impulsive noise [16]. Several sign algorithms (SA) have been proposed in [17-19] to suppress impulsive noise under the assumption of the dense impulse response. In [20], the standard sign least mean square (SLMS) algorithm was proposed in order to achieve the robustness against impulsive noise. For adaptive filters defined in the complex-domain, the Normalized correlation algorithm (NCA) was proposed [21] for robust filtering in severe impulsive noise environments. In [22-24], considering the sparse information in a wireless channel, several sparse SLMS algorithms were proposed to exploit system sparsity and to mitigate non-Gaussian impulsive noise. In [25-26] a flexible zero attractor constraint is utilized in sparse channel estimation under the mixed Gaussian noise environment. However, when the input signal is strongly correlated the performance of sparse SLMS algorithms deteriorates.

When the input to the adaptive filter is assumed to be colored (correlated) input, the standard LMS filter may converge slowly. To improve the filter performance for colored signals, the Affine Projection Algorithm has been proposed [27]. For a large projection order, the APA algorithm has faster convergence, but the steady-state error is higher resulting in a convergence vs steady-state error tradeoff. Also, new sparse algorithms based on Lyapunov stability [28] or reweighted least-mean mixed-norm adaptive filter algorithm [29] for adaptive system identification have been proposed.

In order to utilize the benefits of APA and NCA, the Affine Projection Normalized Correlation Algorithm (AP-NCA) was proposed [30]. The AP-NCA achieves faster convergence for a correlated input and is also robust against impulsive noises. To fully take advantage of the sparse structure present in the system, in this paper, we propose sparse AP-NCA algorithms with different sparse norm constraint functions.

The remaining part of the paper is organized as follows. Section II presents the stochastic models to generate impulse noise and Section III reviews the AP-NCA algorithm. In

Section IV, we propose four sparse AP-NCA algorithms. Simulation results are provided in Section V to validate the effectiveness of the proposed algorithms. Finally, Section VI concludes the paper.

II. IMPULSE NOISE MODELS

The stochastic models used to generate impulse noise are presented in this section. We observe that the two types of impulse noise entering adaptive filtering systems can be the observation noise and another at the input of adaptive filter.

A. Gaussian mixture model (GMM)

A model often used for impulsive observation noise is the Gaussian mixture model (GMM) [31]. GMM is a combination of two independent noise sources $v^{(1)}(n)$ and $v^{(2)}(n)$. The noise source $v^{(1)}(n)$ has a variance $\sigma_{v_1}^2$ with probability of occurrence $(1-\varphi)$, and the noise source $v^{(2)}(n)$ has $\sigma_{v_2}^2$ with the probability of occurrence φ . Usually, $\sigma_{v_2}^2 \gg \sigma_{v_1}^2$. The GMM distribution is given as

$$p(v(n)) = (1-\varphi)N(0, \sigma_{v_1}^2) + \varphi N(0, \sigma_{v_2}^2) \quad (1)$$

The variance of GMM is given by $\sigma_v^2 = E\{v^2(n)\} = (1-\varphi)\sigma_{v_1}^2 + \varphi\sigma_{v_2}^2$. Note that $v(n)$ will reduce to Gaussian noise model if $\varphi = 0$.

B. Bernoulli-Gaussian Model (B-G)

When impulse noise enters the reference input $x(n)$, the filter input $b(n)$ is written as $b(n) = x(n) + q(n)$. $q(n)$ is the impulse noise modeled by a Bernoulli-Gaussian (BG) process [32], given as $q(n) = \alpha(n)v_a(n)$, with $v_a(n)$ assumed to be a White Gaussian process, and its variance is σ_a^2 . $\alpha(n)$ is a binary process, described by the probability $p(\alpha(n) = 1) = P$, $p(\alpha(n) = 0) = 1 - P$, where P represents the probability of occurrence of the impulsive noise, $v_a(n)$.

III. AFFINE PROJECTION NORMALIZED CORRELATION ALGORITHM

The system identification problem is shown in Fig. 1. Let $\bar{x}(n) = [x(n), x(n-1), \dots, x(n-L+1)]^T \in C^{L \times 1}$ be the filter input vector of length L . $x(n)$ is the complex-valued regressor process. The output signal from an unknown system with tap coefficient vector \mathbf{h} is given by $u(n) = \mathbf{h}^T \bar{x}(n)$. $\bar{w}(n) = [w(n), w(n-1), \dots, w(n-L+1)]^T$ is an estimate of \mathbf{h} at iteration n and L is the length of the adaptive filter. The update equation for the APA is given by

$$\bar{w}(n+1) = \bar{w}(n) + \mu \bar{X}(n) [\bar{X}^H(n) \bar{X}(n) + \varepsilon I_M]^{-1} \bar{e}^*(n), \quad (2)$$

where $\bar{X}(n) = [\bar{x}(n), \bar{x}(n-1), \dots, \bar{x}(n-M+1)] \in C^{L \times M}$, is the input signal matrix and M is the projection order. μ is the step-size of APA filter, ε is the regularization term, I_M is the $M \times M$ identity matrix, $(\cdot)^H$ is the conjugate transpose, $(\cdot)^*$ is the complex conjugate, $(\cdot)^T$ is the transpose of a matrix or a vector.

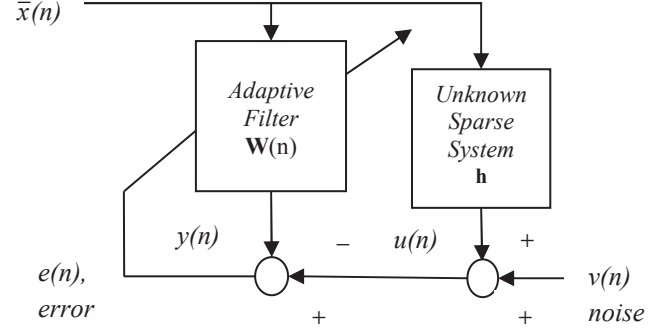


Fig. 1. Block diagram of adaptive sparse system identification

$$\bar{e}^*(n) = \bar{X}^H(n) \boldsymbol{\theta}(n) + \mathbf{v}^*(n), \quad (3)$$

where $\boldsymbol{\theta}(n) = \mathbf{h} - \bar{w}(n)$, is the misalignment vector and,

$\mathbf{v}(n) = [v(n), v(n-1), \dots, v(n-M+1)]^T \in C^{M \times 1}$ is the noise vector. If $M=1$, the APA algorithm simplifies to NLMS algorithm.

The update equation for Normalized Correlation Algorithm (NCA) is given by

$$\bar{w}(n+1) = \bar{w}(n) + \mu \bar{z}(n) / \|\bar{z}(n)\|_1, \quad (4)$$

where, $\bar{z}(n) = [z_0(n), z_1(n), \dots, z_{L-1}(n)]^T = e(n) \bar{x}(n)$ is the correlation vector and $\|\bar{z}(n)\|_1$ is the Euclidean norm of the correlation vector. Since $\|\bar{z}(n)\|_1 = |e(n)| \cdot \|\bar{x}(n)\|$, the update equation can be rewritten as

$$\bar{w}(n+1) = \bar{w}(n) + \mu \varphi_e^*(n) \bar{x}(n) / \|\bar{x}(n)\|_1, \quad (5)$$

with $\varphi_e(n) = e(n) / |e(n)|$.

The Affine Projection Normalized Correlation Algorithm (AP-NCA) is updated as follows

$$\bar{w}(n+1) = \bar{w}(n) + \mu \bar{X}(n) [\bar{X}^H(n) \bar{X}(n) + \varepsilon I_M]^{-1/2} \bar{\varphi}_e^*(n) \quad (6)$$

where, $\bar{\varphi}_e(n) = [\varphi_e(n), \varphi_e(n-1), \dots, \varphi_e(n-M+1)]^T \in \mathbb{C}^{M \times 1}$. If $M=1$, the AP-NCA algorithm behaves as NCA algorithm.

IV. PROPOSED SPARSE ADAPTIVE FILTERING ALGORITHMS

To exploit the system sparsity and robustness against impulsive noises, four sparse algorithms are proposed by introducing effective sparsity constraints into the standard AP-NCA namely, Zero Attracting AP-NCA (ZA-APNCA), Reweighted Zero Attracting AP-NCA (RZA-APNCA), Reweighted L1-norm AP-NCA (RL1-APNCA) and Flexible Zero Attracting AP-NCA (FZA-APNCA).

A. The Zero Attracting AP-NCA (ZA-APNCA) algorithm

Let the cost function of ZA-APNCA algorithm denoted by

$$J_{ZA}(\bar{W}(n)) = J(\bar{W}(n)) + \lambda_{ZA} \|\bar{W}(n)\| \quad (7)$$

where $J(\bar{W}(n))$ is the cost function related to AP-NCA algorithm without sparsity constraint and λ_{ZA} is the regularization parameter which balances the estimation error and $\|\bar{W}(n)\|$.

The weight update equation of ZA-APNCA algorithm is derived as

$$\bar{W}(n+1) = \bar{W}(n) + \mu \bar{X}(n) [\bar{X}^H(n) \bar{X}(n) + \varepsilon I_M]^{-1/2} \bar{\varphi}_e^*(n) - \rho_{ZA} \text{sgn}(\bar{W}(n)), \quad (8)$$

where $\rho_{ZA} = \mu \lambda_{ZA}$ and $\text{sgn}(\cdot)$ denotes the well-known sign function.

B. The Reweighted Zero Attracting AP-NCA (RZA-APNCA) algorithm

Let the cost function of RZA-APNCA algorithm be

$$J_{RZA}(\bar{W}(n)) = J(\bar{W}(n)) + \lambda_{RZA} \sum_{i=0}^{L-1} \log(1 + \varepsilon_{RZA} |w_i(n)|), \quad (9)$$

where λ_{RZA} is the regularization parameter which balances the estimation error and $\sum_{i=0}^{L-1} \log(1 + \varepsilon_{RZA} |w_i(n)|)$.

The weight update equation of RZA-APNCA algorithm is derived as

$$\bar{W}(n+1) = \bar{W}(n) + \mu \bar{X}(n) [\bar{X}^H(n) \bar{X}(n) + \varepsilon I_M]^{-1/2} \bar{\varphi}_e^*(n) - \frac{\rho_{RZA} \text{sgn}(\bar{W}(n))}{1 + \varepsilon_{RZA} |\bar{W}(n)|} \quad (10)$$

where $\rho_{RZA} = \mu \lambda_{RZA} \varepsilon_{RZA}$

C. The Reweighted L1-norm AP-NCA (RL1-APNCA) algorithm

Let the cost function of RL1-APNCA algorithm be

$$J_{RL1}(\bar{W}(n)) = J(\bar{W}(n)) + \lambda_{RL1} \|\bar{f}(n) \bar{W}(n)\| \quad (11)$$

where λ_{RL1} is the weight associated with the penalty term and

$$[\bar{f}(n)]_i = \frac{1}{\delta_{RL1} + |\bar{W}(n-1)_i|}, \quad i = 0, 1, \dots, L-1 \quad (12)$$

$\delta_{RL1} > 0$ and hence $[\bar{f}(n)]_i > 0$ for $i = 0, 1, \dots, L-1$.

The weight update equation of RL1-APNCA algorithm is derived as

$$\bar{W}(n+1) = \bar{W}(n) + \mu \bar{X}(n) [\bar{X}^H(n) \bar{X}(n) + \varepsilon I_M]^{-1/2} \bar{\varphi}_e^*(n) - \frac{\rho_{RL1} \text{sgn}(\bar{W}(n))}{\delta_{RL1} + |\bar{W}(n-1)|}, \quad (13)$$

where $\rho_{RL1} = \mu \lambda_{RL1}$.

D. The Flexible Zero Attracting AP-NCA (FZA-APNCA) algorithm

The flexible zero attractor is realized using the approximation parameter adjustment function defined as

$$S_\alpha(\bar{W}(n)) = (1 + \alpha^{-1}) \left(1 - e^{-\alpha |\bar{W}(n)|}\right), \quad (14)$$

where α is a small positive constant.

The modified cost function obtained by incorporating $S_\alpha(\bar{W}(n))$ function into the AP-NCA cost function is the following

$$J_{FZA}(\bar{W}(n)) = J(\bar{W}(n)) + \rho_{FZA} S_\alpha(\bar{W}(n)). \quad (15)$$

The weight update equation of FZA-APNCA algorithm is derived as

$$\bar{W}(n+1) = \bar{W}(n) + \mu \bar{X}(n) [\bar{X}^H(n) \bar{X}(n) + \varepsilon I_M]^{-1/2} \bar{\varphi}_e^*(n) - \rho_{FZA} S'_\alpha(\bar{W}(n)), \quad (16)$$

where,

$$S'_\alpha(\bar{W}(n+1)) = (\alpha + 1) e^{(-\alpha |\bar{W}(n+1)|)} \text{sgn}(\bar{W}(n+1)) \quad (17)$$

V. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed sparse adaptive algorithms in the context of system identification. The length of the unknown system is set as $L = 16$ with system sparsity of $K = \{4, 8\}$ and the adaptive filter is also assumed to have the same length. The correlated (colored) input signal is generated by using a Gaussian white noise with variance $\sigma_x^2 = 1$ (0 dB) through a first-order autoregressive process, AR(1), with a 0.5 pole. The system noise $v(n)$ contains white Gaussian noise with SNR = 20 dB and impulse noise. The algorithms are compared based on the performance of the Mean Square Error (MSE) between the actual and estimated CIR. The average of 100 trials is used in evaluating the results.

Detailed parameters for computer simulation are listed in Table I.

TABLE I. SIMULATION PARAMETERS

Parameters	Values
Input Signal	Correlated/Colored Input: AR(1) Gaussian process with pole 0.5; $x(n)=0.5x(n-1)+u(n)$
Unknown System Length	$L=16$
No. of nonzero coefficients	System sparsity, $K=\{4, 8\}$
Distribution of nonzero coefficients	Random Gaussian distribution $N(0,1)$
Projection order	$M=4$
SNR	20 dB
Noise types	Case 1: "white" Gaussian noise, $\sigma_v^2 = 0.01$ (-20 dB) Case 2: Observation noise: Gaussian Mixture Model (GMM) $\phi = 0.1$, $\sigma_{v_1}^2 = 0.01$ (-20 dB), $\sigma_{v_2}^2 = 10$ (10 dB). Case 3: Impulse noise at filter input: Bernoulli-Gaussian (B-G) model $p_{v_a} = 0.1$, $\sigma_{v_a}^2 = 1000$ (30 dB) Case 4: GMM & impulse noise at filter input

A. Comparison of the proposed sparse AP-NCA algorithms under noise case 1

The performance of the proposed sparse algorithms under the assumption of "white" Gaussian noise is shown in Fig. 2. It can be noticed that the proposed sparse APNCA algorithms exhibit better performance in terms of MSE when the system is highly sparse and it reduces as the system sparsity increases. The FZA-APNCA algorithms achieve minimum steady state error value.

B. Comparison of the proposed sparse AP-NCA algorithms under noise case 2

The performance of the proposed sparse algorithms under the assumption of GMM modeled impulsive observation noise is shown in Fig. 3. It can be noticed that the proposed sparse APNCA algorithms exhibit better performance in terms of MSE when the system is highly sparse and it reduces as the system sparsity increases. The FZA-APNCA achieves minimum steady state error value.

C. Comparison of the proposed sparse AP-NCA algorithms under noise case 3

The performance of the proposed sparse algorithms under the assumption of impulse noise at filter input is shown in Fig. 4. It can be noticed that the proposed sparse APNCA algorithms exhibit better performance in terms of MSE when the system is highly sparse and it reduces as the system sparsity increases.

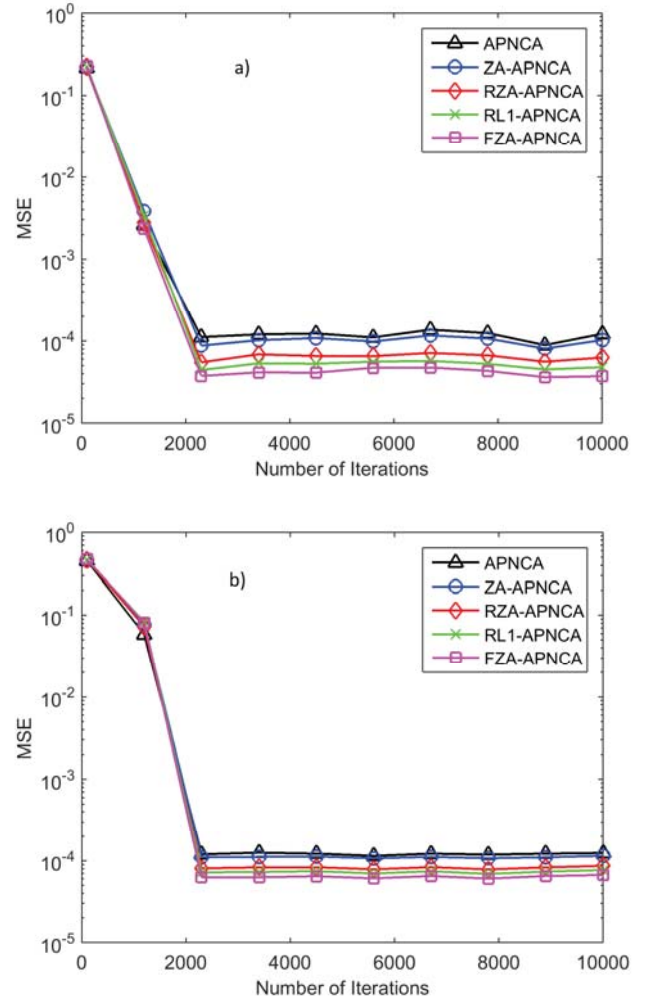


Fig. 2. MSEs of the proposed sparse AP-NCA algorithms for noise case 1 ("white" Gaussian) with the projection order, $M=4$ and different system sparsity of, (a) $K=4$, and (b) $K=8$.

D. Comparison of the proposed sparse AP-NCA algorithms under noise case 4

The performance of the proposed sparse algorithms under the assumption of GMM observation noise & impulse noise at filter input is shown in Fig. 5. It can be noticed that the proposed sparse APNCA algorithms exhibit better performance in terms of MSE when the system is highly sparse and it reduces as the system sparsity increases. The FZA-APNCA achieves minimum steady state error value. Similar results were obtained for higher L values in all previous cases. Our future work will be focused on theoretical convergence analysis and examining the tracking abilities of the proposed algorithms.

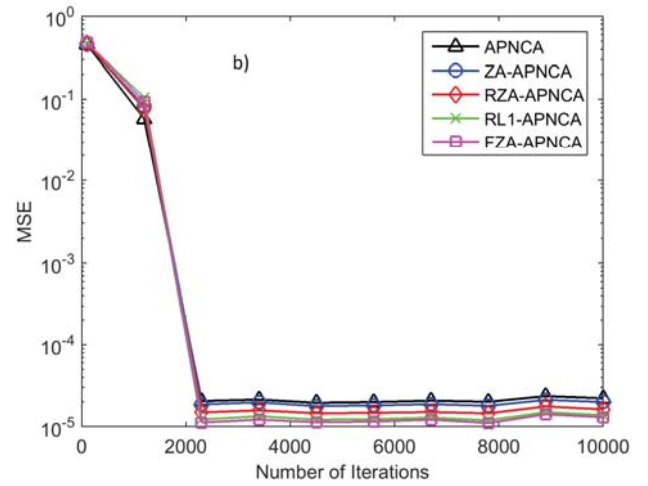
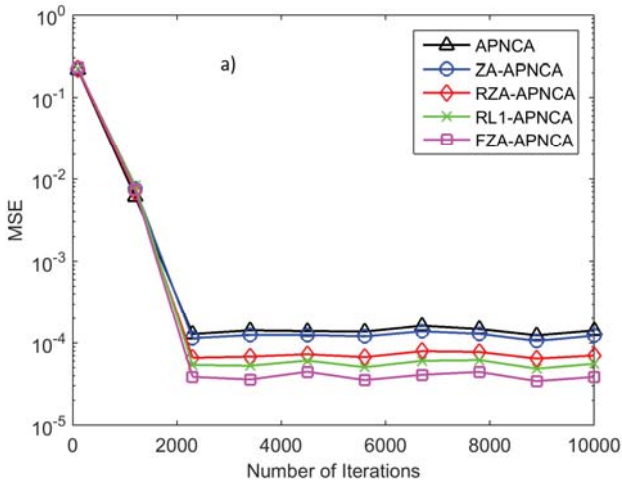


Fig. 4. MSEs of the proposed sparse AP-NCA algorithms for noise case 3 (impulse noise at filter input: B-G) with the projection order, $M=4$ and different system sparsity of, (a) $K=4$ and (b) $K=8$.

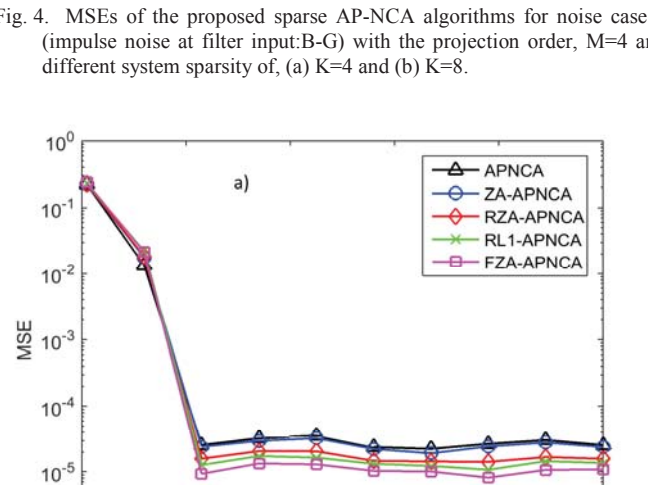
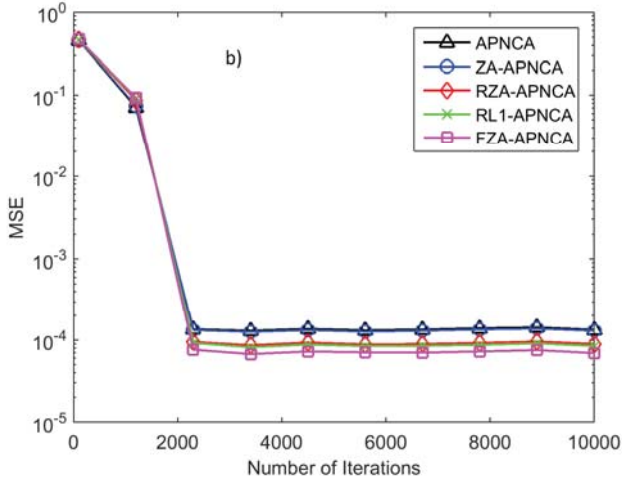


Fig. 3. MSEs of the proposed sparse AP-NCA algorithms for noise case 2 (impulsive observation noise: GMM) with the projection order, $M=4$ and different system sparsity of, (a) $K=4$, and (b) $K=8$.

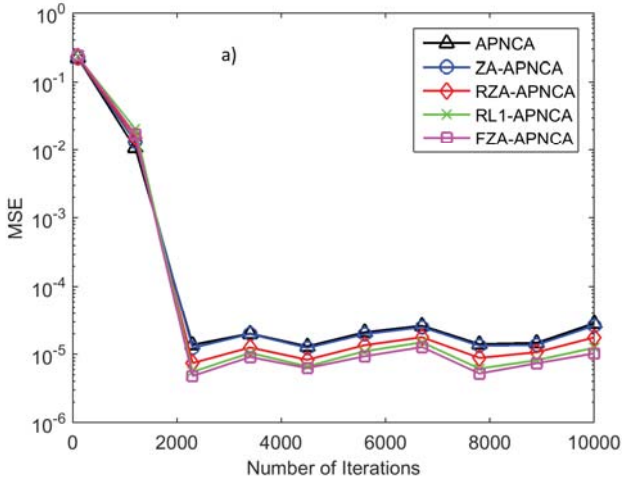
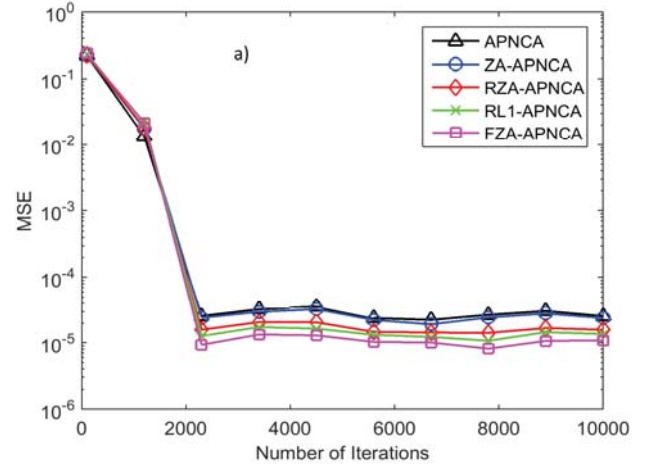


Fig. 5. MSEs of the proposed sparse AP-NCA algorithms for noise case 4 (GMM noise & impulse noise at filter input) with the projection order, $M=4$ and different system sparsity of, (a) $K=4$ and (b) $K=8$.

VI. CONCLUSION

The AP-NCA algorithm developed for adaptive filters in the complex domain has faster convergence for correlated inputs and at the same time highly robust in the presence of impulsive noise, but it does not promote sparsity. Hence, in this paper, we have proposed four sparse APNCA algorithms in the sparse system identification context. Simulation results validate our proposed sparse algorithms in exploiting the system sparsity as well as robust to impulsive observation noise and impulsive filter input in the complex domain. Moreover, the proposed FZA-APNCA algorithm exhibit superior performance in Gaussian and non-Gaussian noise environments.

ACKNOWLEDGMENT

The work of Felix Albu was supported by a grant from the Romanian National Authority for Scientific research and Innovation, CNCS/CCCDI-UEFISCDI project number PN-III-P2-2.1-PED-2016-0651.

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