Approximated Proportionate Affine Projection Algorithms for Block-Sparse Identification

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***Abstract*— In this paper two block-sparse approximated memory improved proportionate affine projection algorithm are proposed for block sparse system identification. An approximation is used for a recently proposed family of block-sparse proportionate affine projection algorithms. It is shown that the proposed algorithms have close convergence performance to the original ones and they are less numerically complex. An investigation of the influence of their parameters is also presented.**

***Keywords*— Proportionate affine projection algorithm; block-sparse system identification; adaptive filter.**

I. INTRODUCTION

The adaptive filtering [1] has found application in many areas such as active noise control [2-3], image processing [4], echo cancellation [5] etc. The proportionate affine projection algorithms (PAPAs) provide good performance for applications dealing with long and sparse echo paths. Unlike the normalized least mean square (NLMS) [1] and the affine projection algorithm (APA) [6], the family of proportionate algorithms exploits this sparseness. Each coefficient of the adaptive filter is updated independently of the others and the adaptation step-size is adjusted depending on its magnitude. The proportionate APA (PAPA) [7], the memory improved PAP algorithm (MIPAPA) [8], the μ-law MIPAPA (MMIPAPA) [9] algorithms were developed starting from this principle. The main difference between the above mentioned algorithms is given by the proportionate matrix computation. An approximated MIPAPA (AMIPAPA) was proposed in [10] in order to reduce the complexity of MIPAPA.

Recently, the block-sparse PNLMS (BS-PNLMS) algorithm was proposed for identifying block-sparse systems [11]. The same main idea led also to designing a family of block-sparse PAPAs for block-sparse system identification [12]. It was shown in [12] that the PNLMS, BS-PNLMS, APA and PAPA algorithms are special cases of block-sparse PAPA (BS-PAPA).

In this paper, a fast version of AMIPAPA-family type algorithms is developed by combining the proportionate matrix approximation from [10] with an adaptation of block-sparse memory IPAPA (BS-MIPAPA) [12].

The paper is organized as follows. Section II presents the proposed algorithm and investigates its efficient implementation leading to a reduced numerical complexity. The simulation results are presented in Section III. Finally, the conclusions are given and ideas for further improvements are proposed.

II. THE PROPOSED ALGORITHMS

In the echo cancellation applications, an adaptive filter is used to model the echo path [1]. The far-end signal is $x(n)$, where n is the time index and the reference signal of the adaptive filter, $d(n)$, contains the output of the echo path i.e., the echo signal and the near-end signal.

$$d(n) = \mathbf{h}^T(n)\mathbf{x}(n) + v(n), \tag{1}$$

where $\mathbf{x}(n) = [x(n), x(n-1), ..., x(n-L+1)]^T$, $\mathbf{h}$(n) are the unknown coefficients, $v(n)$ is the measurement noise and L is the length of the impulse response.

The estimated error vector is given by

$$\mathbf{e}(n) = \mathbf{d}(n) - \mathbf{X}^T(n)\hat{\mathbf{h}}(n-1), \tag{2}$$

Where $\mathbf{d}(n) = [d(n), d(n-1), ..., d(n-M+1)]$, M is the projection order, $\mathbf{e}(n) = [e(n), e(n-1), ..., e(n-M+1)]$ with $e(n) = d(n) - \mathbf{x}^T(n)\hat{\mathbf{h}}(n-1)$, $\hat{\mathbf{h}}(n)$ is the adaptive filter's coefficients and $\mathbf{X}(n) = [\mathbf{x}(n), \mathbf{x}(n-1), ..., \mathbf{x}(n-M+1)]$.

The block-sparse scheme for BS-PAPA was derived in [12] and it is based on the optimization of the $l_{2,1}$ norm. The $l_{2,1}$ norm was defined as [12]

$$\left\|\hat{\mathbf{h}}\right\|_{2,1} = \sum_{i=1}^{N} \sqrt{\hat{\mathbf{h}}_{[i]}^T \hat{\mathbf{h}}_{[i]}}, \tag{3}$$

where

$$\hat{\mathbf{h}}_{[i]} = \left[\hat{h}_{(i-1)P+1}, \hat{h}_{(i-1)P+2}, ..., \hat{h}_{iP}\right]^T \tag{4}$$

P is a group size parameter and $N = L/P$ [12].

The update equation for BS-MIPAPA is

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$$\hat{\mathbf{h}}(n)=\hat{\mathbf{h}}(n-1)+\mu\mathbf{P}'(n)\hat{\boldsymbol{\varepsilon}}(n) \quad (5)$$

where $\hat{\boldsymbol{\varepsilon}}(n)$ is the LDL^T method [13] solution of the $\mathbf{S}'(n)\hat{\boldsymbol{\varepsilon}}(n)=\mathbf{e}(n)$ system. The matrix is obtained as follows

$$\mathbf{S}'(n)=\delta\mathbf{I}_M+\mathbf{X}^T(n)\mathbf{P}'(n), \quad (6)$$

$$\mathbf{P}'(n)=\left[\mathbf{g}(n-1)\odot\mathbf{x}(n) \quad \mathbf{P}'_{-1}(n-1)\right], \quad (7)$$

where δ is a regularization term, $\mathbf{I}_M$ is an identity matrix, the matrix

$$\mathbf{P}'_{-1}(n-1)=\begin{bmatrix}\mathbf{g}(n-2)\odot\mathbf{x}(n-1) \\ ..\mathbf{g}(n-M)\odot\mathbf{x}(n-M+1)\end{bmatrix} \quad (8)$$

contains the first $M-1$ columns of $\mathbf{P}'(n-1)$ and the operator $\odot$ denotes the Hadamard product [8]. The proportionate matrix is computed as follows [12]

$$\mathbf{G}(n-1)=diag\left[g_1(n-1)\mathbf{1}_P,...,g_N(n-1)\mathbf{1}_P\right]. \quad (9)$$

where $\mathbf{1}_P$ is the P-length row vector of all ones and

$$g_l(n-1)=\frac{1-\alpha}{2L}+\frac{(1+\alpha)\left\|\hat{\mathbf{h}}_l(n-1)\right\|_2}{2\sum_{i=0}^{N-1}\left\|\hat{\mathbf{h}}_i(n-1)\right\|_2+\xi}, \quad (10)$$

with $-1\le\alpha<1$ and ξ is a small positive constant.

Using a similar approach as in [10], important computational savings can be obtained if an approximation is made to $\mathbf{S}'(n)$. A symmetric matrix, $\mathbf{S}''(n)$, is obtained by updating both its first row and its first column with $\mathbf{X}^T(n)\cdot\left[\mathbf{g}(n-1)\odot\mathbf{x}(n)\right]$ and adding δ to the first element [10]. The bottom-right $(M-1)\times(M-1)$ submatrix of $\mathbf{S}''(n)$ is replaced with the top-left $(M-1)\times(M-1)$ submatrix of $\mathbf{S}''(n-1)$. The resulted algorithm, called the block sparse approximated MIPAPA (BS-AMIPAPA), will have the following update

$$\hat{\mathbf{h}}(n)=\hat{\mathbf{h}}(n-1)+\mu\mathbf{P}'(n)\tilde{\boldsymbol{\varepsilon}}(n), \quad (11)$$

where $\tilde{\boldsymbol{\varepsilon}}(n)$ is the solution of the $\mathbf{S}''(n)\tilde{\boldsymbol{\varepsilon}}(n)=\mathbf{e}(n)$ linear system.

The logarithmic proportionate updating scheme [9] can be easily incorporated in BS-AMIPAPA by modifying the proportionate coefficients of (10) as in (12)-(13):

$$g_l(n-1)=\frac{1-\alpha}{2L}+\frac{(1+\alpha)\mathrm{F}\left(\left\|\hat{\mathbf{h}}_l(n-1)\right\|_2\right)}{2\sum_{i=0}^{N-1}\mathrm{F}\left(\left\|\hat{\mathbf{h}}_i(n-1)\right\|_2\right)+\xi} \quad (12)$$

$$\mathrm{F}\left(\left\|\hat{\mathbf{h}}_l(n-1)\right\|_2\right)=\ln\left(1+\mu_1\left\|\hat{\mathbf{h}}_l(n-1)\right\|_2\right) \quad (13)$$

where μ_1 is a constant. The BS-AMMIPAPA requires additional N logarithmic operations and N additions per iteration in comparison with the BS-AMIPAPA. This additional increase in complexity is moderate and does not depend on the projection order. The numerical complexity in terms of multiplications of the investigated algorithms is the following:

$$C_{\text{BS-MIPAPA}}=L(4M+1+1/P)+M+C_m, \quad (14)$$

$$C_{\text{BS-AMIPAPA}}=L(3M+2+1/P)+M+C_m \quad (15)$$

where C_m indicates the numerical complexity in terms of multiplications associated with solving the linear systems of equations using the LDL^T method ([13], [15]). Figure 1 shows the numerical complexity comparison for two cases: fixed, variable M (L = 1024) and variable L (M = 8) respectively. It can be seen from Fig. 1 that BS-AMIPAPA is less complex in terms of multiplications than BS-MIPAPA, especially for large filter lengths or projection orders.

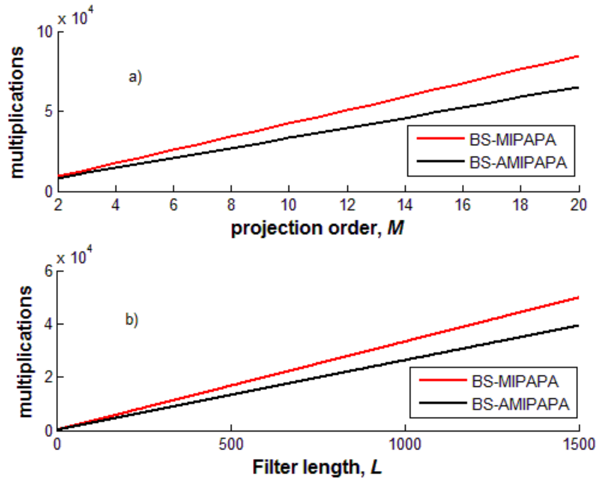

Fig. 1. Numerical complexity of the considered algorithms in terms of multiplications for $P=16$ in two situations: a) variable M; b) variable L.

For example, when P = 16, L = 1024, M = 8, BS-MIPAPA needs 34004 multiplications per iteration, while BS-AMIPAPA needs 26836 multiplications per iteration (i.e., it is around 25% less complex).

III. Simulation Results

The performance of the proposed BS-AMIPAPA and BS-AMMIPAPA is evaluated via simulations. The length of the unknown system is L = 1024, and an exact modeling is assumed. Two block-sparse impulse systems in Fig. 2 are used [12]. The first impulse response in Fig. 2a has a single cluster of nonzero coefficients at [257, 288]. The two clusters in the second impulse response in Fig. 2b are located at [257, 288] and [769, 800] separately. All the path bursts have 32 samples. In order to compare the tracking ability for different algorithms, an echo path change was incurred at 30000-sample by switching from the first impulse response in Fig. 2a to the second impulse response in Fig. 2b [12].

The algorithms were tested using colored noise which was generated by filtering white Gaussian noise (WGN) through a first order system with a pole at 0.8 [12]. Independent WGN is added to the system background with SNR = 30dB. The projection order was M = 8, and the step-sizes was $\mu = 0.01$ [12]. The regularization parameters δ was set to 0.01, $\rho = 0.01$, $q = 0.01$ and $\mu_1 = 10$. The convergence state of adaptive filter is evaluated with the normalized misalignment which is defined as $20\log_{10}(\|\mathbf{h} - \hat{\mathbf{h}}(n)\|_2/\|\mathbf{h}\|_2)$. For the proposed algorithms the regularization stabilization technique from [14] is used. We found in our simulations that the error norm between the vectors containing the first row using the updating procedure of BS-AMIPAPA and BS-MIPAPA for various P values were smaller than 0.001. This indicates that the approximation used for BS-AMIPAPA and BS-AMMIPAPA is justified.

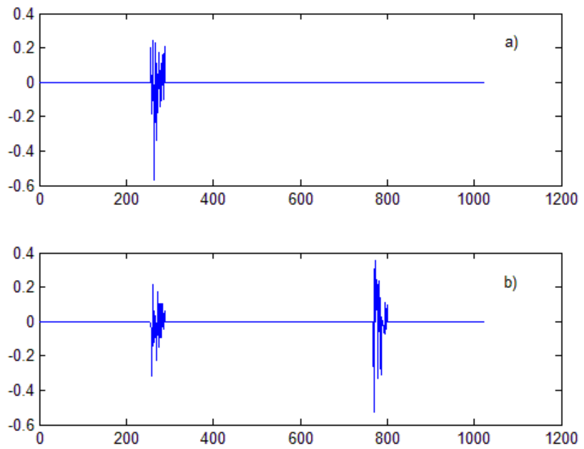

Fig. 2 Block-sparse impulse systems a) one-cluster block-sparse system, b) two-cluster block-sparse system [8].

Figure 3 shows that this fact leads to a close performance of the proposed algorithms to that of the original algorithms. It can be noticed that the normalized misalignment difference between the original algorithms and the approximated ones is smaller than 0.4 dB. Figure 3 also shows that the normalized misalignment difference is much smaller for the two-cluster block-sparse system than for one-cluster block-sparse system.

The influence of μ_1 parameter of BS-AMMIPAPA is examined in Fig. 4. It can be seen that the best performance is obtained for $\mu_1 = 10$ and this value is used for the next simulations.

In Fig. 5, the performance of the proposed BS-AMIPAPA and BS-MMIPAPA are compared for different group sizes P chosen as 16, 32 and 128. The impact of different group sizes on BS-AMIPAPA and BS-AMMIPAPA is similar with the best performance being obtained for P = 32 for BS-AMIPAPA and P = 16 for BS-AMMIPAPA.

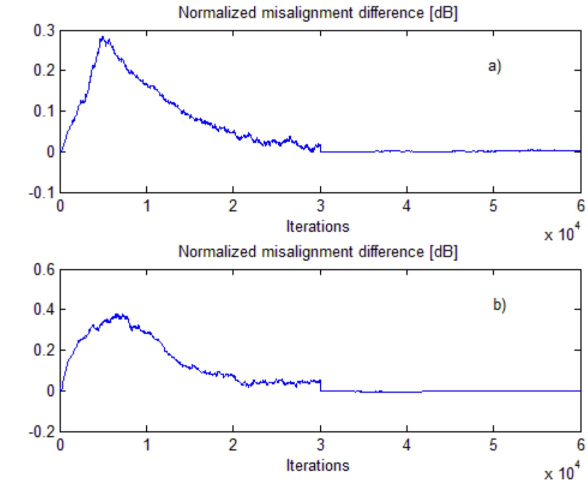

Fig. 3. The normalized misalignment difference between: a) BS-AMIPAPA and BS-MIPAPA; b) BS-AMMIPAPA and BS-MMIPAPA

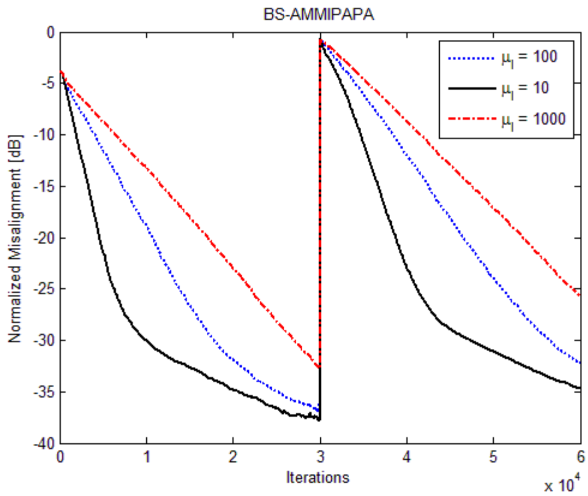

Fig. 4. The normalized misalignment of BS-AMMIPAPA for different μ_1 values, colored input and SNR = 30 dB.

As discussed in [11] and [12] the group size of BS-PNLMS, BS-PAPA and BS-MPAPA should be chosen properly in order to fully exploit the block-sparse characteristic. Our simulations have shown that the BS-AMIPAPA and BS-AMMIPAPA achieves similar convergence behavior for properly chosen parameters (see Fig. 5).

In the last simulation, we compare the performance of BS-AMIPAPA and BS-AMMIPAPA algorithms together with APA, PAPA and BS-PAPA [12]. For BS-PAPA, BS-AMIPAPA and BS-AMMIPAPA algorithms, the group size was P = 32. The convergence curves for colored input with SNR = 30 dB are shown in Fig. 6.

It can be seen that both proposed BS-AMIPAPA and BS-AMMIPAPA outperform PAPA and APA in terms of convergence speed and tracking ability.

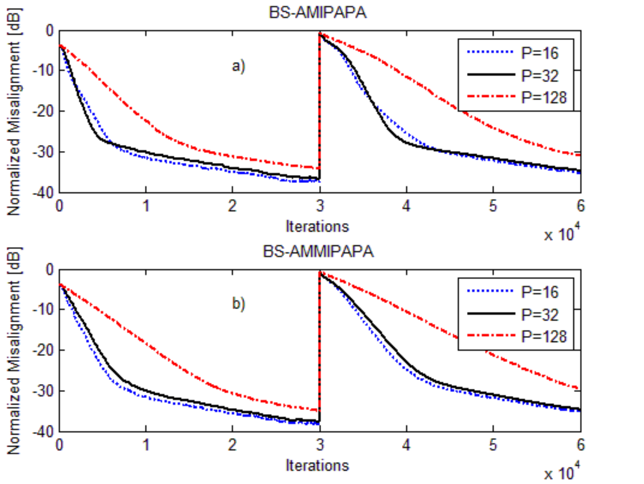

Fig. 5. The normalized misalignment for different group sizes, colored input and SNR = 30 dB: a) BS-AMIPAPA; b) BS-AMMIPAPA.

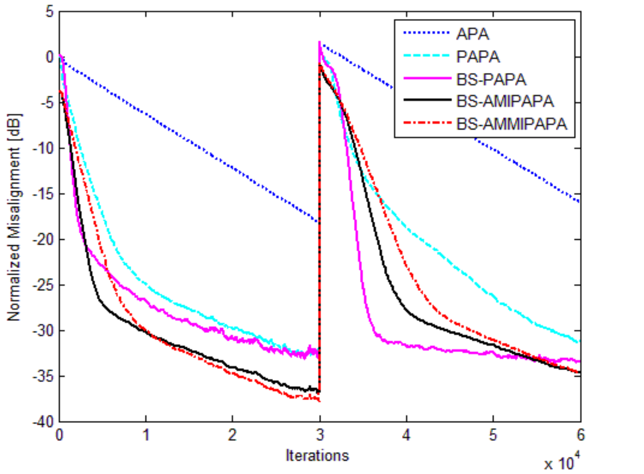

Fig. 6. Comparison of APA, PAPA, BS-PAPA, BS-AMIPAPA and BS-AMMIPAPA for colored noise with SNR = 30dB.

The proposed algorithms provide better convergence performance than BS-PAPA for the one-cluster sparse system, while providing a good compromise for the two-cluster sparse system. Future work will be focused on developing fast recursive versions as in [14], exploiting sparsity as in [16] and investigating their suitability for adaptive feedback cancellation [17-18].

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