2017 21st International Conference on System Theory, Control and Computing (ICSTCC)

The l_p- norm Proportionate Normalized Least Mean Square Algorithm for Active Noise Control

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***Abstract*—The l_p norm-constrained proportionate normalized least-mean-square (LP-PNLMS) using the modified filtered-x structure is proposed for active noise control. It is shown that better performance is obtained for primary and secondary paths having a wide range of sparseness levels when compared with competing sparsity-inducing algorithms at a price of moderate complexity increase.**

Keywords— active noise control; adaptive algorithms; l_p norm;

I. INTRODUCTION

Active noise control (ANC) techniques using adaptive filters have been widely used for removing noise [1]. The ANC algorithms takes into account the secondary path that causes delays in signal transmission [2]. The filtered-x (Fx) scheme using the least-mean square (LMS) algorithm has been introduced in [2]. Better performance was reported by the more complex affine projection algorithms or its variants (e.g. [3] - [6]). One of the shortcomings of the Fx scheme is the slow convergence of the adaptive filter caused by the necessity of using small step sizes [3] - [6]. The modified filtered-x (MFx) approach [7] greatly improved the convergence speed with the penalty of an increased numerical complexity due to an additional filtering step. Numerous algorithms have been proposed using the MFx scheme (e.g. [8] - [10]).

In many cases, the system to be identified presents a degree of sparsity [3]. Several algorithms incorporating sparsity penalties were proposed, e.g. the zero-attracting (ZA) and reweighted zero-attracting (RZA) algorithms [11], [12]. The corresponding ZA version of the NLMS algorithm can be easily obtained by setting the projection order to one in the ZA-MFxAP algorithm from [12]. The numerical complexity of the ZA and RZA versions for ANC is too high and a simpler option that has a faster convergence than LMS or NLMS algorithms for sparse primary or secondary ANC paths is to use the proportionate NLMS (PNLMS) algorithm [13] or the related μ - law PNLMS algorithm [14]. The combination of improved PNLMS (IPNLMS) filters for active noise control was proposed in [15] - [16]. Unfortunately, the numerical complexity of this approach is at least double. A simpler alternative called zero attracting PNLMS (ZA-PNLMS) has been proposed in [17].

Recently, the l_p-norm-constrained proportionate normalized least-mean-square (LP-PNLMS) algorithm has been proposed [18]. It was shown that the incorporation of the weighted l_p-norm and the PNLMS approach was leading to an increased convergence performance for a sparse channel estimation application [18].

In this paper we incorporate the LP-PNLMS algorithm into the MFx structure and investigate its use for different primary and secondary paths with various sparseness levels. The influence of two parameters of the modified filtered-x LP-PNLMS (MFx-LPPNLMS) algorithm is investigated and its performance is compared with that of competing algorithms for an ANC application. To the best of our knowledge, the application of the MFx versions of the PNLMS and its l_p-norm version has not been investigated yet. Also, we show that, when ZA-PNLMS is used in a MFx scheme, it is a particularization of our MFx-LPPNLMS algorithm, having inferior performance in several ANC situations. Its behavior for an active noise control application has not been investigated in [17].

Section II presents the ANC system and the equations of MFx-LPPNLMS algorithm. In Section III, the proposed algorithm is compared with competing ANC algorithms. In Section IV several conclusions and further direction of research are presented.

II. THE PROPOSED ALGORITHM

In broadband feedforward ANC, the noise is reduced by subtracting from the acoustic signal a generated signal by using an error signal [1]. In the MFx structure shown in Fig. 1, $\mathbf{q}(k)$ is the primary path, and the instantaneous error signal $\hat{e}(k)$ is estimated [7]. The signal $x(k)$ and the estimated secondary path, $\hat{\mathbf{s}}(k)$, are needed to generate the $x_f(k)$ signal. In this structure it is not needed for the signal $d(k)=\mathbf{x}(k)\mathbf{h}(k+1)$ to be available, where $\mathbf{x}(k)$ collects L consecutive samples of $x(k)$, L being the filter length [11].

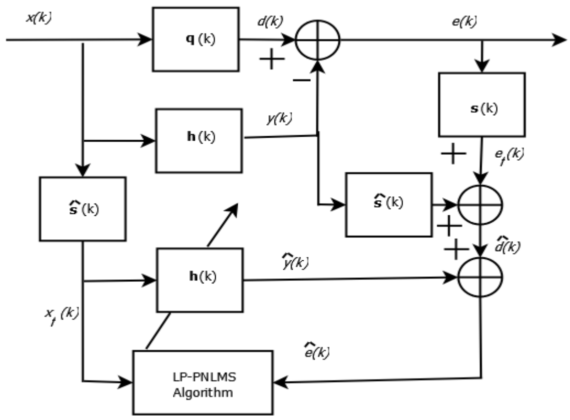

Fig. 1: The MFX structure using the proposed LP-PNLMS algorithm.

The condition $\hat{d}(k)=\mathbf{x}_f(k)\mathbf{h}(k+1)$ is imposed where $\mathbf{x}_f(k)$ collects L consecutive samples of $x_f(k)$ [11]. We also have $\hat{y}(k)=\mathbf{x}_f(k)\mathbf{h}(k)$.

A. The MFX-PNLMS algorithm

The PNLMS algorithm uses a proportionate technique, where the coefficients with a higher magnitude have a larger step-size importance, and therefore, these active taps converge faster. The MFx-PNLMS algorithm update equation with reference to Fig. 1 is the following:

$$\mathbf{h}(k+1)=\mathbf{h}(k)+\frac{\mu_{PNLMS}\mathbf{C}(k)\hat{e}(k)}{\delta_{PNLMS}+\mathbf{x}_f^T(k)\mathbf{C}(k)} \quad (1)$$

where $\delta_{PNLMS}=\sigma_{x_f}^2/L$ is a regularization factor, μ_{PNLMS} is the step size and $\mathbf{C}(k)=\mathbf{G}(k)\mathbf{x}_f(k)$ where

$$\mathbf{G}(k)=diag\left(g_0(k),g_1(k),...,g_{L-1}(k)\right), \quad (2)$$

with each $g_i(k)$ computed as follows

$$g_i(k)=\frac{\chi_i(k)}{\sum_{i=0}^{L-1}\chi_i(k)}, \quad 0\le i\le L-1 \quad (3)$$

with

$$\chi_i(k)=\max\left[\rho_g\ \max\left[\delta_p,\left|h_0(k)\right|,...,\left|h_{L-1}(k)\right|,\left|h_i(k)\right|\right]\right], \quad (4)$$

where ρ_g and δ_p are small positive constants.

B. The MFX-LPPNLMS algorithm

The LP-PNLMS algorithm [18] incorporates the l_p-norm into the PNLMS cost function and uses the gain-matrix weighted l_p-norm in designing the zero attractor. As shown in [18] the following relation is obtained:

$$\begin{aligned}\mathbf{h}(k+1)=&\ \mathbf{h}(k)+\frac{\mu_{LP}\hat{e}(k)\mathbf{C}(k)}{\varepsilon_{LP}+\mathbf{x}_f^T(k)\mathbf{C}(k)}-\\&-\left\{\mathbf{I}-\mathbf{B}(k)\right\}\frac{\varepsilon_{LP}\left\|\mathbf{h}(k)\right\|_p^{1-p}\operatorname{sgn}\left(\mathbf{h}(k)\right)}{\varepsilon+\left|\mathbf{h}(k)\right|^{1-p}},\end{aligned} \quad (5)$$

where

$$\mathbf{B}(k)=\mathbf{C}(k)\mathbf{x}_f^T(k)\left\{\mathbf{x}_f^T(k)\mathbf{C}(k)\right\}^{-1}, \quad (6)$$

μ_{LP} is a step size, $\varepsilon_{LP}=\sigma_{x_f}^2/L$, γ_{LP} and ε are small constants, $\left\|\mathbf{h}\right\|_p=\left(\sum_i h_i^p\right)^{1/p}$ is the p-norm of $\mathbf{h}$, $0\le p\le 1$. It is assumed that the elements of $\mathbf{B}(k)$ are smaller than one as in [17] or [18]. This assumption also leads to an important complexity reduction of the proposed MFx-LPPNLMS algorithm whose update equation with reference to Fig. 1 is given by:

$$\begin{aligned}\mathbf{h}(k+1)=&\ \mathbf{h}(k)+\frac{\mu_{LP}\hat{e}(k)\mathbf{C}(k)}{\varepsilon_{LP}+\mathbf{x}_f^T(k)\mathbf{C}(k)}\\&-\rho_{LP}\frac{\left\|\mathbf{h}(k)\right\|_p^{1-p}\operatorname{sgn}\left(\mathbf{h}(k)\right)}{\varepsilon+\left|\mathbf{h}(k)\right|^{1-p}},\end{aligned} \quad (7)$$

where $\rho_{LP}=\mu_{LP}\gamma_{LP}$. It should be noted that the zero-attracting PNLMS (ZA-PNLMS) algorithm proposed in [17] if applied using a MFx scheme is a simple particularization of the MFx-LPPNLMS algorithm for p = 1. Its behavior for an active noise control application has not been investigated in [17]. By comparing Eq. (1) with Eq. (7), the additional complexity of MFx-LPPNLMS over MFx-PNLMS is given by the l_p-norm term. Besides the additional L multiplications and L divisions, there is a moderate complexity increase associated with computing L values of $\left\|\mathbf{h}(k)\right\|_p^{1-p}$ and $\left|\mathbf{h}(k)\right|^{1-p}$ respectively.

III. Simulation results

This section presents the simulation results of the MFx-LPPNLMS, MFx-PNLMS and MFx-ZANLMS [12] algorithms for an ANC application using the same primary and secondary paths as in [11]. For all the algorithms, the parameters were tuned to the same values as in [11]. For each primary path the algorithms were run for 45,000 iterations with the secondary path set as sparse at the start of the experiment, changed to partially-sparse at iteration 5,000 and to non-sparse at iteration 25,000. For the simulations the following parameters were used: $\delta = 0.002$, $\delta_p = 0.01$, $\varepsilon = 0.05$, $\rho_g = 0.00625$ and L=800. The performance of the algorithms has been plotted using the mean-square deviation (MSD) convergence curves. The step sizes were adjusted to obtain the same initial convergence speed for each ANC situation. For the sparse primary path case, the step size values of the MFx-LPPNLMS algorithm, for each secondary path in the sequence shown above were 1, 0.6 and 0.4 respectively. For the semi-sparse primary path, the step size values of the MFx-LPPNLMS for each secondary path were 1, 0.6 and 0.9 respectively. Finally, for the non-sparse primary path, the step size values for each secondary path were 1, 1 and 0.7 respectively. These values and other parameters were similar with those used in [11] and [12] and facilitates the comparison with MFx-ZANLMS. In the next simulation examples, the parameter value that provide the best performance compromise is the value that has the smallest MSD average over all 45000 iterations.

Figure 2 shows a comparison of MFx-LPPNLMS for different ρ_{LP} values for a sparse plant. It can be noticed that the convergence speed of the MFx-LPPNLMS algorithm is decreasing when ρ_{LP} value is decreased from 10^{-4} to 10^{-9}. However, the best compromise is obtained for $\rho_{LP} = 10^{-5}$ if the average steady-state MSD performance for all cases is taken into account as mentioned above.

In Fig. 3, the MSD performance for the same ρ_{LP} values as above and the semi-sparse path case is shown. For this case, the best compromise regarding the convergence speed and overall performance for all the secondary paths is obtained for $\rho_{LP} = 10^{-9}$.

In Fig. 4 the MSD performance for various ρ_{LP} values and the non-sparse path case is shown. In this case, the best overall performance compromise regarding the convergence speed is obtained for $\rho_{LP} = 10^{-8}$.

Figure 5 examines the MSD performance for a sparse plant and variable p from 0.5 to 1 with 0.1 increment.

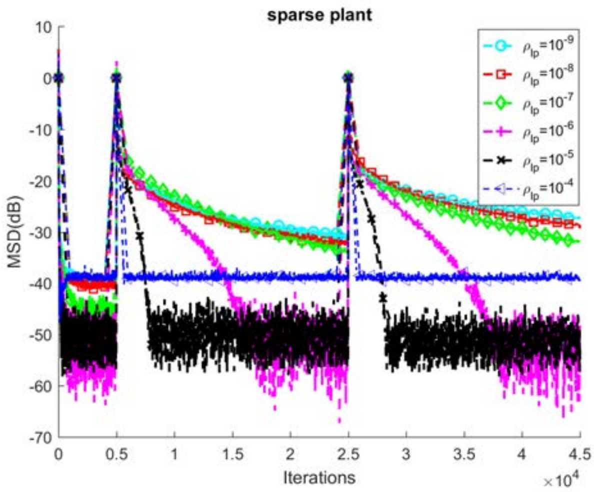

Fig. 2. MSD results of MFx-LPPNLMS algorithm for various ρ_{LP} values and a sparse plant.

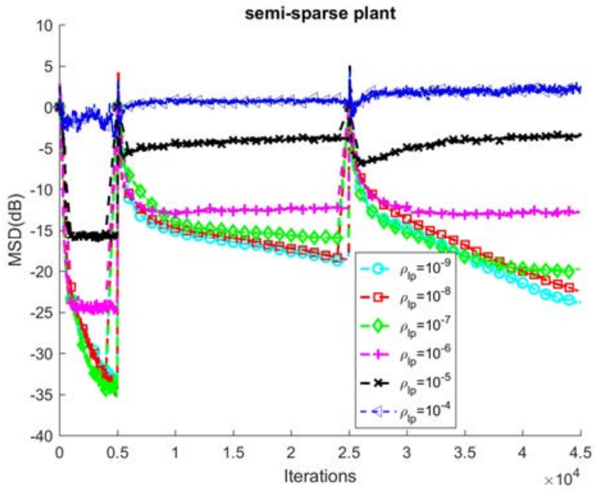

Fig. 3. MSD results of MFx-LPPNLMS algorithm for various ρ_{LP} values and a semi-sparse plant.

It is found that p = 0.7 is the value that provides the best compromise for the sparse plant case.

Figure 6 shows the MSD performance for a semi-sparse plant. It is found that the best compromise is obtained for p = 0.5.

The same conclusion regarding the value of p = 0.9 can be found and noticed from Fig. 7 where the MSD performance for a non-sparse plant is shown. Figures 8-10 shows the MSD performance comparison of MFx-LPPNLMS, MFx-PNLMS and MFX-ZANLMS for sparse, semi-sparse and non-sparse plants respectively.

Therefore, it is obvious that a careful selection of the ρ_{LP} and p parameters should be made for the best convergence performance of the proposed MFx-LPPNLMS algorithm for plants with various sparseness values. The parameters chosen for the MFx-LPPNLMS for each plant are those identified above as providing the best performance compromise. It can be easily noticed that the proposed algorithm achieves the best results for the sparse primary plant and sparse secondary plant.

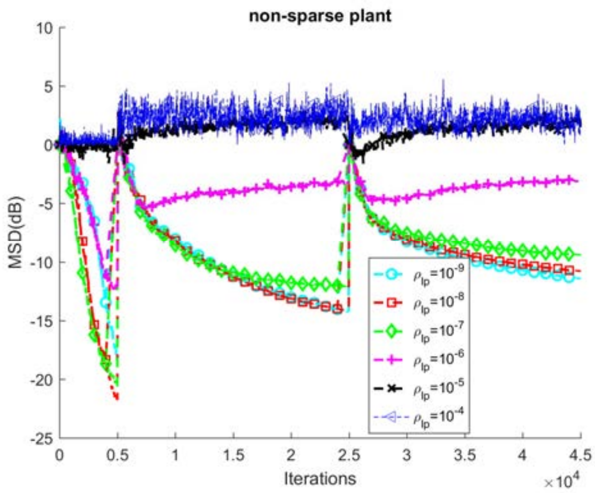

Fig. 4. MSD results of MFx-LPPNLMS algorithm for various ρ_{LP} values and a non-sparse plant.

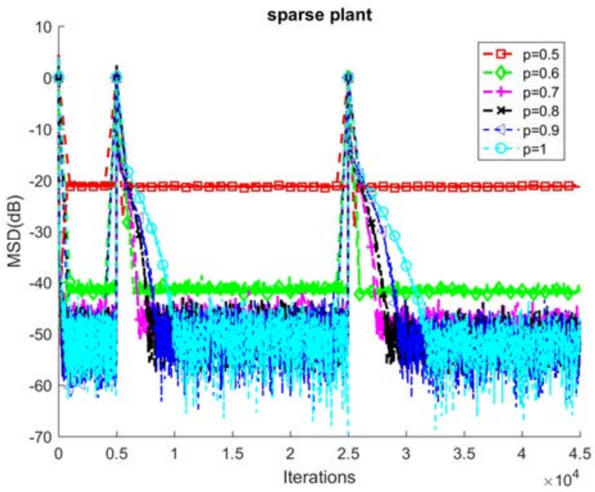

Fig. 5. MSD results of MFx-LPPNLMS algorithm for various *p* values and a sparse plant.

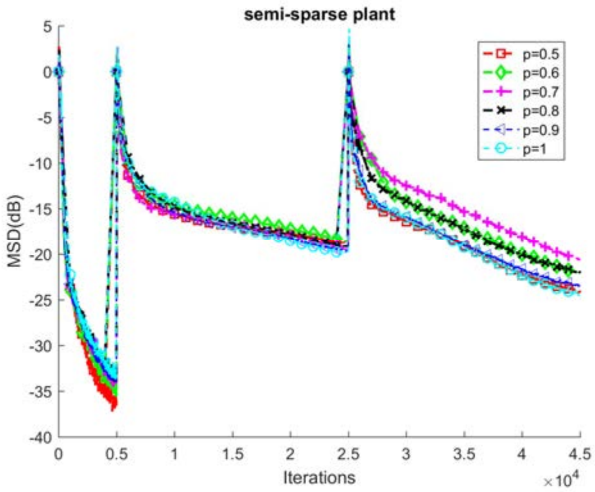

Fig. 6. MSD results of MFx-LPPNLMS algorithm for various *p* values and a semi-sparse plant.

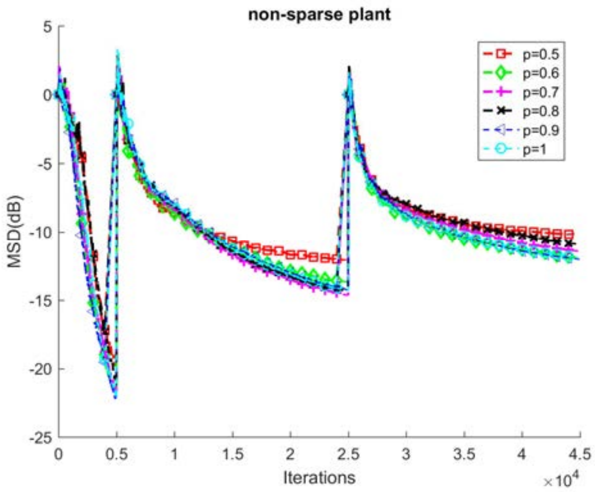

Fig. 7. MSD results of MFx-LPNLMS algorithm for various *p* values and a non-sparse plant.

For the semi-sparse plant, the proposed algorithm provides the best results among the considered algorithms for the sparse secondary path.

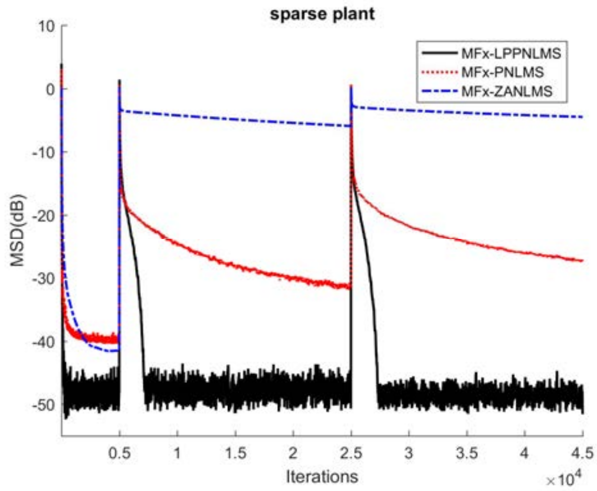

Fig. 8. MSD results of MFx-LPPNLMS, MFx-PNLMS and MFx-ZANLMS algorithms for a sparse plant.

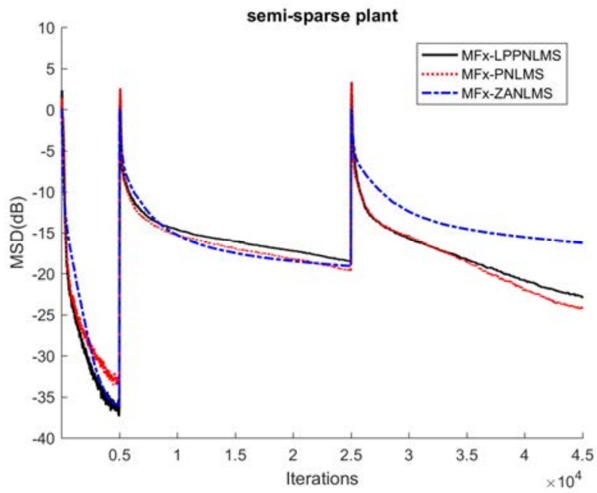

Fig. 9. MSD results of MFx-LPPNLMS, MFx-PNLMS and MFx-ZANLMS algorithms for a semi-sparse plant

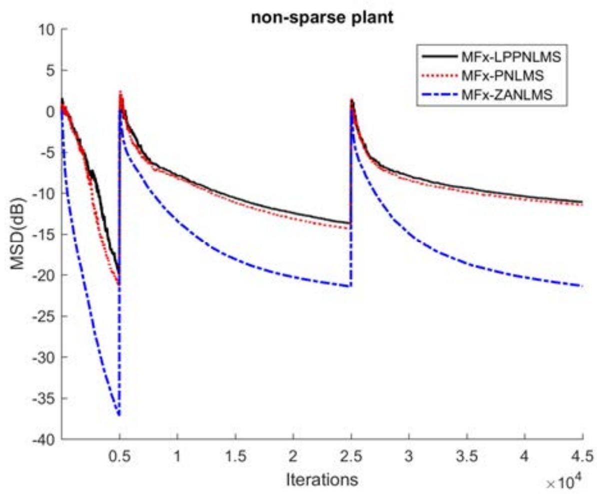

Fig. 10. MSD results of MFx-LPPNLMS, MFx-PNLMS and MFx-ZANLMS algorithms for a non-sparse plant.

Alternatively, the MFX-ZANLMS algorithm has the best MSD results in case of non-sparse plant. This confirms the numerous studies that have shown the weakness of the proportionate algorithms for identifying dispersive systems [13], [15]. Therefore, the MFx-LPPNLMS algorithm, with a proper parameter selection can represent a practical alternative for ANC, especially for sparse and semi-sparse plants.

Further work might incorporate variable step-size/projection order [9], use the correntropy criterion [20] or reweighted least-mean mixed-norm approach [21] in order to develop new MFx-based active noise control algorithms and investigate their real-time performance.

IV. Conclusions

This paper has proposed the MFx-LPPNLMS algorithm for ANC systems. The simulation results show that the MFx-LPPNLMS algorithm provides MSD improvements over competing algorithms for sparse and semi-sparse plants in case of a wide sparseness range of secondary paths. These performance improvements are obtained with a moderate complexity increase. Therefore, the MFx-LPPNLMS algorithm can be a good option for ANC systems.

Acknowledgment

This work was supported by a grant of the Romanian National Authority for Scientific research and Innovation, CNCS/CCCDI-UEFISCDI project number PN-III-P2-2.1-PED-2016-0651. This work was also partially supported by the National Key Research and Development Program of China-Government Corporation Special Program (2016YFE0111100), the Science and Technology Innovative Talents Foundation of Harbin (2016RAXXJ044), Projects for the Selected Returned Overseas Chinese Scholars of Heilongjiang Province and MOHRSS of China, and the Foundational Research Funds for the Central Universities (HEUCFD1433 and HEUCF160815).

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