



ANALYSIS OF LNS IMPLEMENTATION OF THE QRD-LSL ALGORITHMS

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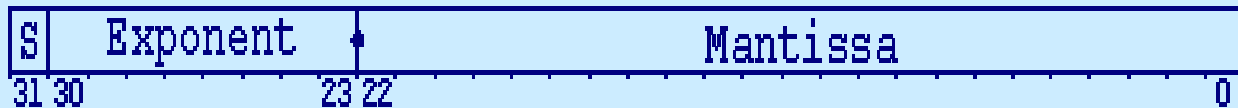
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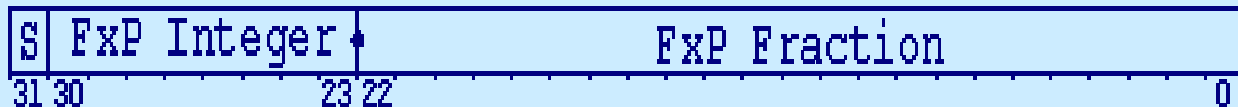


Logarithmic number system

IEEE Single Precision:



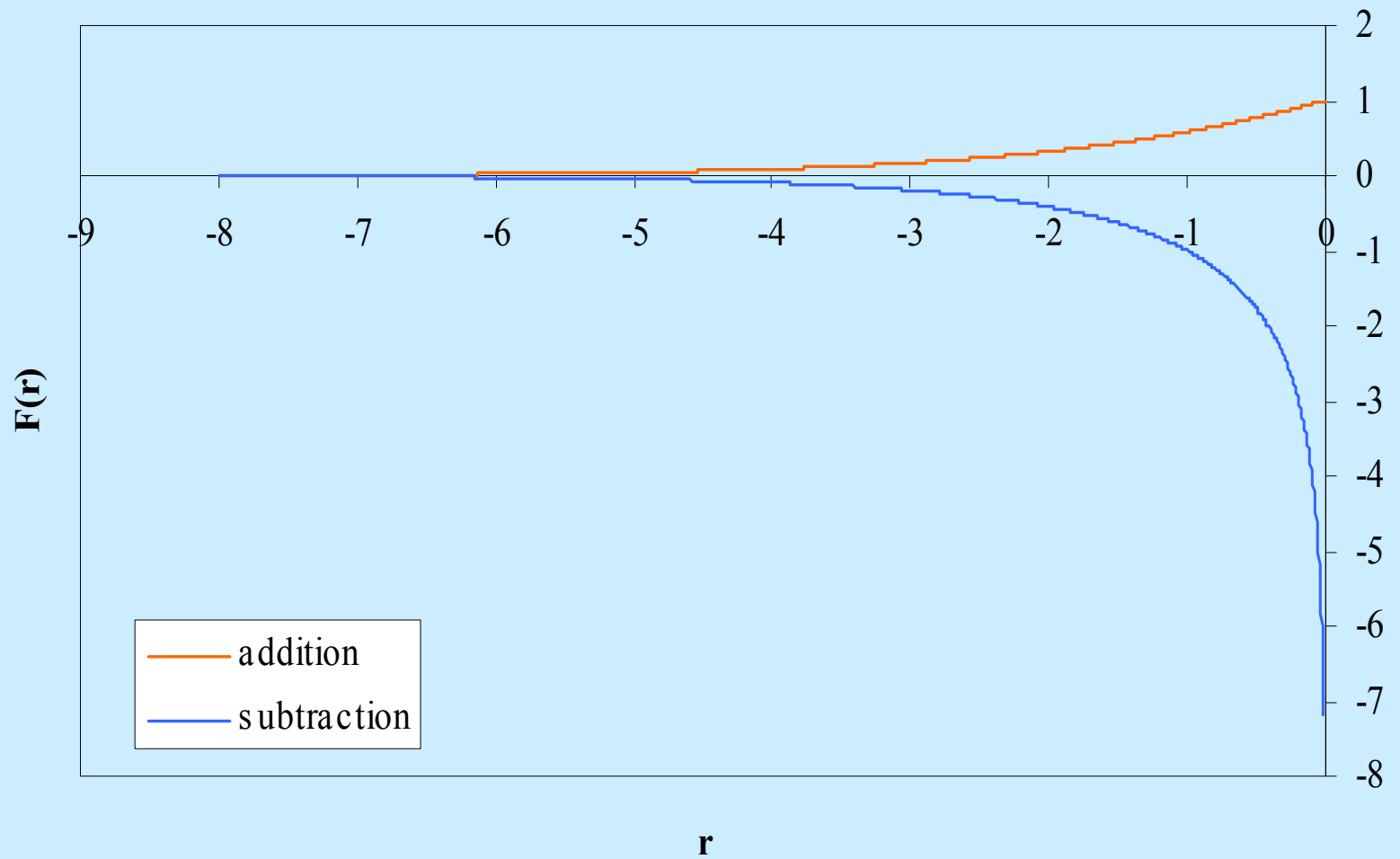
32b LNS:



IEEE standard single precision floating point representation
and the 32-bit LNS format



Logarithmic number system





Logarithmic number system

$x + y$	ADD	$Lz = Lx + \log(1+2^{(Ly-Lx)})$, Sz depends on sizes of x,y
$x - y$	SUB	$Lz = Lx + \log(1-2^{(Ly-Lx)})$, Sz depends on sizes of x,y
$x * y$	MUL	$Lz = Lx + Ly$, Sz = Sx OR Sy
x / y	DIV	$Lz = Lx - Ly$, Sz = Sx OR Sy
x^2	SQU	$Lx \ll 1$, Sz = Sx
$x^{0.5}$	SQRT	$Lx \gg 1$, Sz = Sx
x^{-1}	RECIP	$Lz = Lx$, Sz = -Sx
$x^{-0.5}$	RSQRT	$Lz = Lx \gg 1$, Sz = -Sx

LNS Arithmetic Operations



MQRD-LSL Algorithm

Adaptive Algorithm	QRD-LSL	MQRD-LSL
Multiplications	$25M+11$	$22M+10$
Divisions	$4M+2$	$4M+2$
Additions/subtractions	$8M+3$	$8M+3$
Square-root operations	$4M+2$	0

Computational complexities (number of operations per iteration) for the QRD-LSL algorithms.



MQRD-LSL Algorithm

Assumptions

$$\begin{aligned}
 r_{m-1}(n-1) &= \sqrt{\frac{\beta_{f,m-1}(n-1)}{\beta_{b,m-1}(n-1)}} = \sqrt{1 + \frac{\beta_{f,m-1}(n-1) - \beta_{b,m-1}(n-1)}{\beta_{b,m-1}(n-1)}} \cong \\
 &\cong 1 + \frac{1}{2} \cdot \frac{\beta_{f,m-1}(n-1) - \beta_{b,m-1}(n-1)}{\beta_{b,m-1}(n-1)} = \\
 &= \frac{1}{2} + \frac{1}{2} \cdot \frac{\beta_{f,m-1}(n-1)}{\beta_{b,m-1}(n-1)}
 \end{aligned}$$

$$\frac{\beta_{f,m-1}(n-1) - \beta_{b,m-1}(n-1)}{\beta_{b,m-1}(n-1)} \ll 1$$

$$\frac{|e_{m-1}^f(n)|^2}{\sum_{i=1}^{n-1} |e_{m-1}^f(i)|^2} \ll 1$$

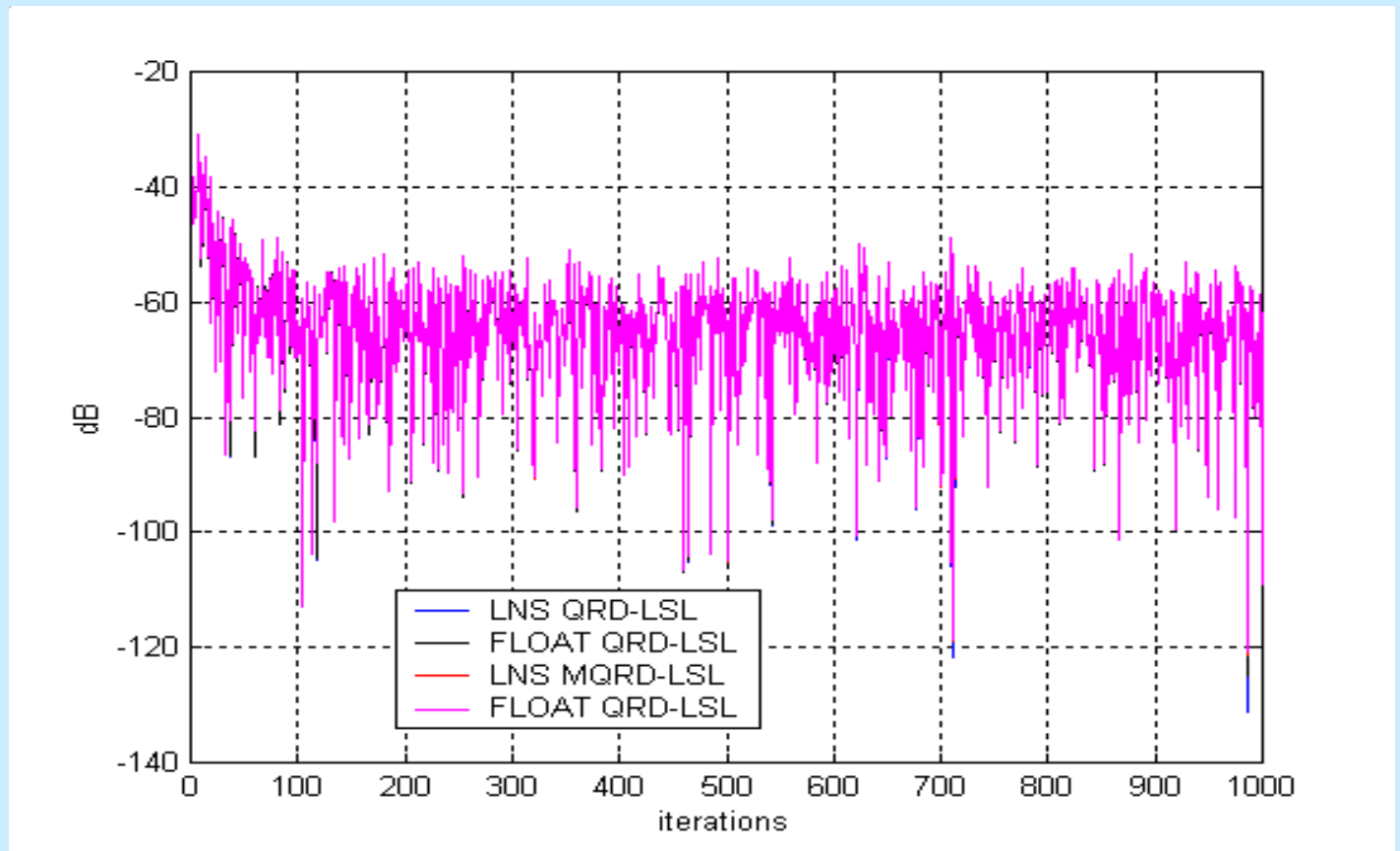
$$\frac{|e_{m-1}^b(n-1)|^2}{\sum_{i=1}^{n-2} |e_{m-1}^b(i)|^2} \ll 1$$

$$\sum_{i=1}^n \lambda^{n-i} f(i) \cong \sum_{i=1}^n f(i)$$



Simulations

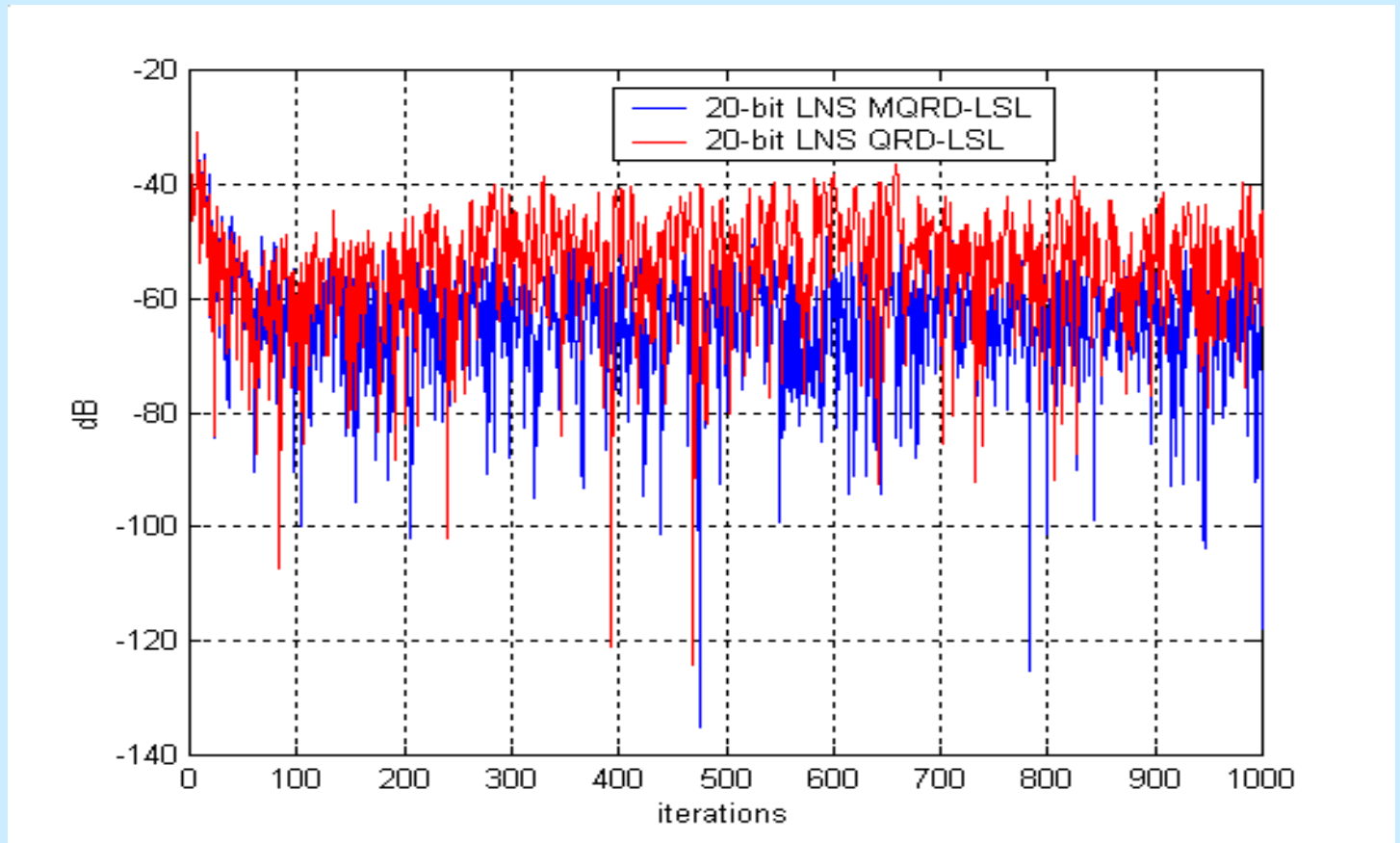
The square errors for 32-bit FLOAT and 32-bit LNS implementations of MQRD-LSL and QRD-LSL algorithms





Simulations

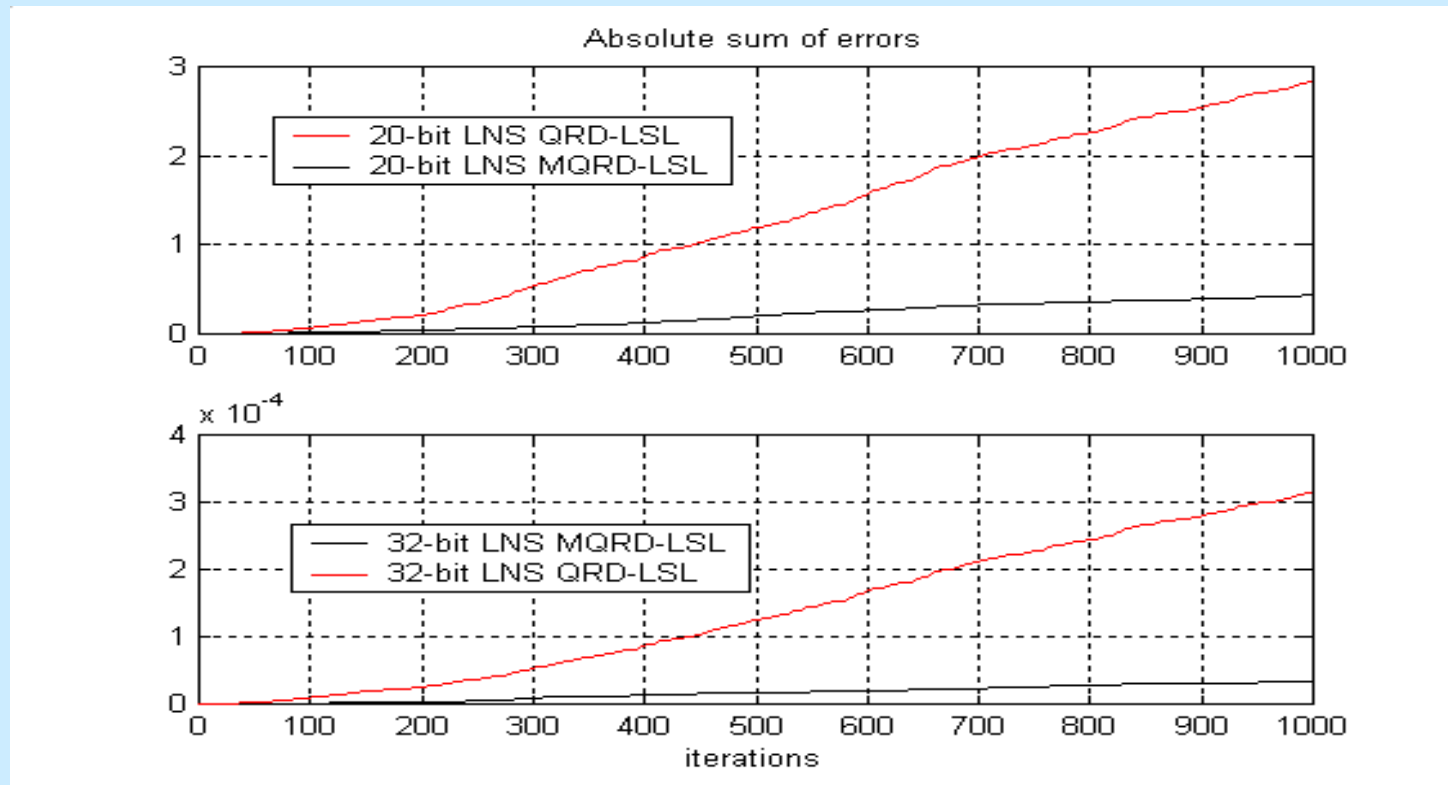
The square errors for 20-bit LNS implementations of the classical and modified QRD-LSL algorithm





Simulations

The absolute sum of errors for 20-bit LNS (up) and 32-bit FLOAT or LNS implementations (down) of the QRD-LSL Algorithms

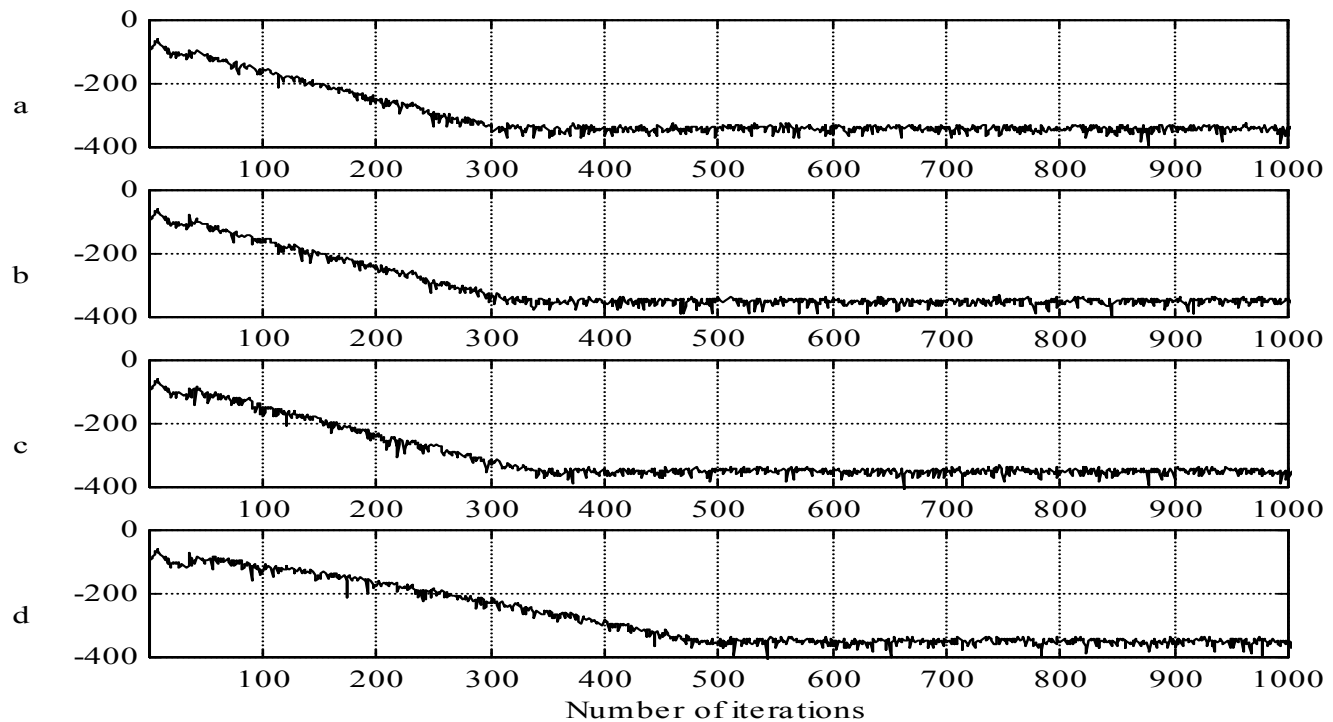




Simulations

Square errors [dB], $\lambda=0.9$

(a) QRD-LSL classical algorithm, (b) QRD-LSL-v1, (c) QRD-LSL-v2, (d) MQRD-LSL

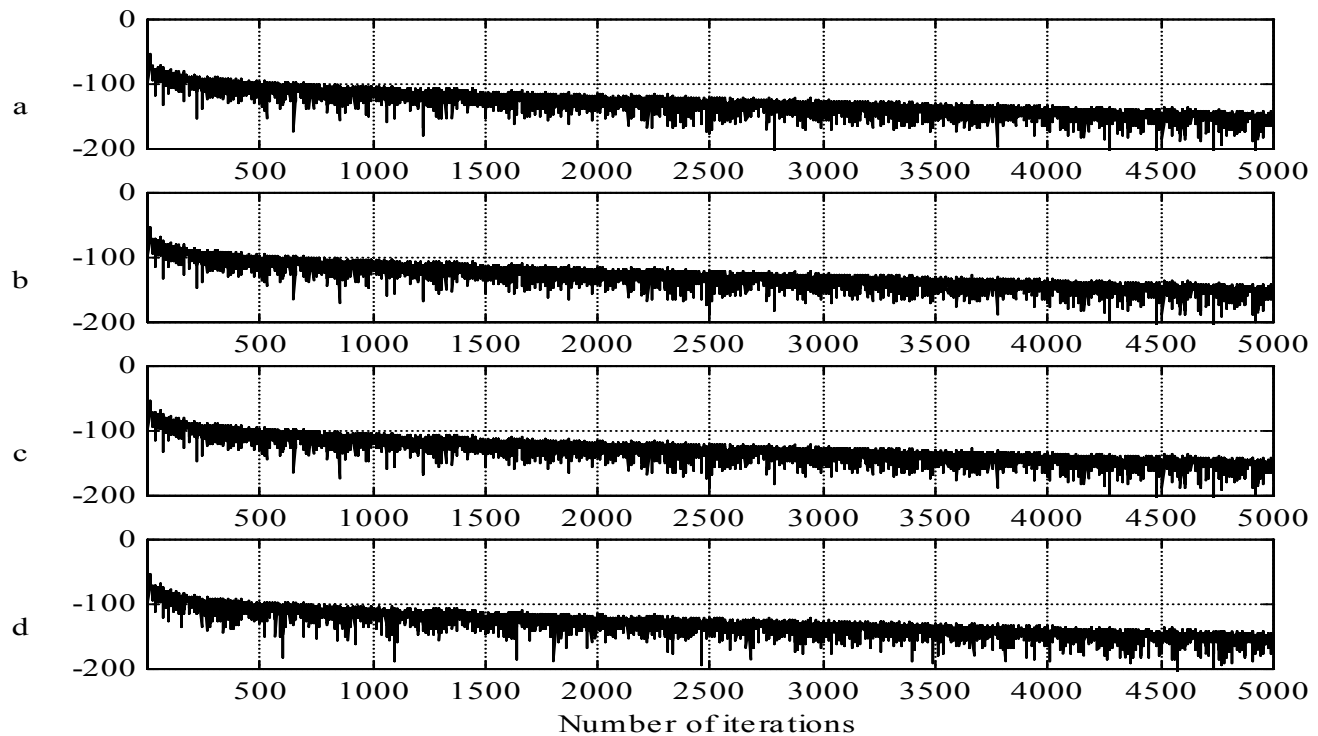




Simulations

Square errors [dB], $\lambda=0.999$

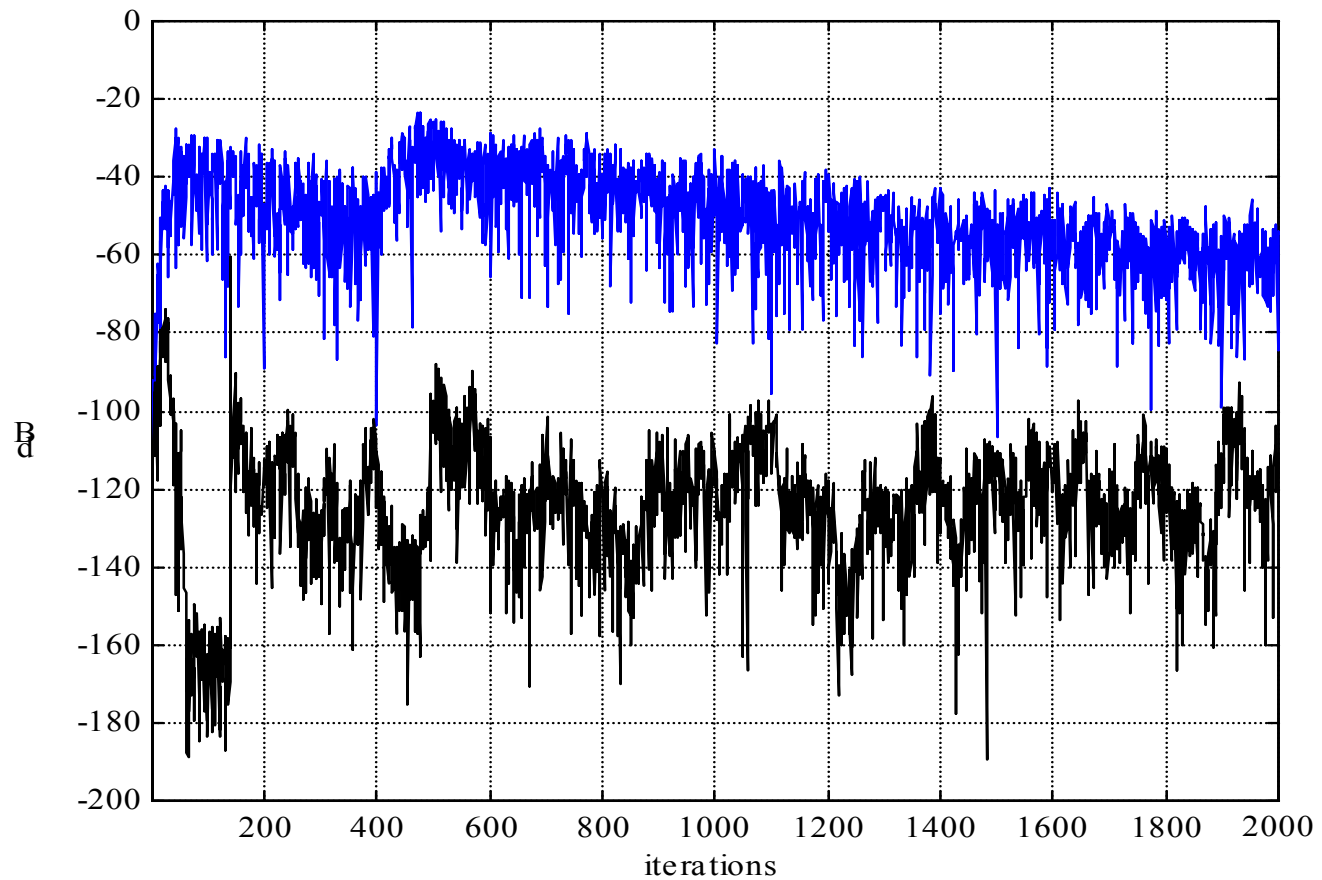
(a) QRD-LSL classical algorithm, (b) QRD-LSL-v1, (c) QRD-LSL-v2, (d) MQRD-LSL





Simulations

Square errors for the NLMS and modified QRD-LSL algorithms





Conclusions

- The QRD-LSL algorithms have efficient LNS implementations because of their numerous divisions or square-root operations that are very time consuming in floating-point implementations.
- The QRD-LSL algorithms are attractive because of their fast convergence rate even for high length filters and high correlated signals.
- The proposed MQRD-LSL algorithm eliminates the square root operations and achieves a lower computational complexity than the classical QRD-LSL algorithm.
- We showed in this paper that the 20-bit LNS implementation of the MQRD-LSL algorithms offers better performances than the 20-bit LNS implementation of the classical QRD-LSL algorithm.



Questions ?

HSLA Project webpage

<http://napier.ncl.ac.uk/HSLA>