Modified Least-Mean Mixed-Norm Algorithms For Adaptive Sparse System Identification Under Impulsive Noise Environment

Pogula Rakesh, T. Kishore Kumar
Department of Electronics and Communication Engineering
National Institute of Technology Warangal
Warangal, India
Email: rakesh.pogula453@gmail.com,
kishorefr@gmail.com

Felix Albu
Department of Electronics
Valahia University of Targoviste
Targoviste, Romania
Email: felix.albu@valahia.ro

***Abstract*—In this paper, new algorithms robust to a mix of Gaussian and impulsive noises that approximate an unknown sparse impulse response of an LTI system are proposed. They are using the sigmoid cost function and based on the Least-Mean Mixed-Norm (LMMN) adaptive algorithm. It is shown by simulations that the proposed sigmoid LMMN (SLMMN) algorithms that exploit sparsity-enforcing penalties achieve superior performance to other competing algorithms in the sparse system identification context.**

Keywords—adaptive algorithm; impulsive noise; least-mean mixed-norm; sigmoid function; sparse system identification

I. Introduction

There are numerous applications of the adaptive filters [1]-[3]. One of the most popular algorithms used for system identification is the least mean square (LMS) algorithm [4] due to its computational simplicity. In [5], the least mean fourth (LMF) algorithm based on fourth-order power optimization criterion was first proposed. To overcome the sensitivity issues of LMS and LMF, the least-mean mixed-norm (LMMN) algorithm which is a linear combination of LMS and LMF is proposed in [6]. However, the performance of LMMN algorithm degrades seriously due to impulsive interferences which exist in practical environments [7]-[9]. Typical impulsive noises are the following noises: computer keyboard clicks, switching noise, audio recordings dropouts [10].

Adaptive algorithms based on lower-order norms (l_p-norm) [9], [11] and the family of sign algorithms (SAs) [12], [13] have become popular due to their simplicity and robustness against impulsive noise. The least mean p-power (LMP) algorithm minimizing the l_p-norm of the error signal has been successfully employed for system identification under impulsive noise disturbances [14], [15]. In [16], robust mixed-norm (RMN) algorithm is also developed. However, these algorithms suffer from slow convergence rate. Other algorithms robust to such noises are proposed in [17], [18]. Recent studies focus on the nonlinear sigmoid function which can be used in the traditional cost function of the adaptive filtering algorithms to improve the robustness to impulsive noise [19]-[21].

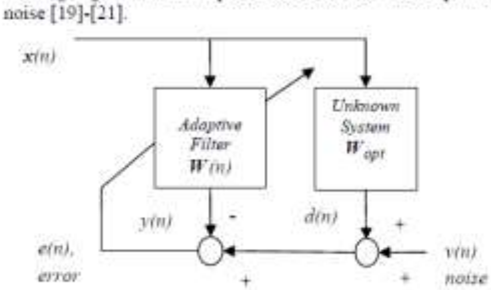

Fig. 1. System identification model

In this paper, we propose a modified LMMN algorithm based on the sigmoid cost function. This new algorithm is called Sigmoid LMMN (SLMMN) which can improve the estimation performance under impulsive noise.

Usually, the impulse response of many systems encountered in practice exhibit sparse structure, i.e., they have very few non-zero taps among many inactive ones [22]. The reweighted least-mean mixed-norm algorithm has been recently proposed for sparse channel estimation under Gaussian noise assumption [23]. In [24], sparse LMP algorithms are applied for robust estimation of sparse channels. Unfortunately, the proposed SLMMN algorithm cannot utilize the a priori sparse structure of the system. Hence, two sparsity aware algorithms namely, Zero Attracting (ZA)-SLMMN and Reweighted Zero Attracting (RZA)-SLMMN are proposed to exploit the system sparsity under impulsive noise environment. The general update rule using the gradient descent is used for all the investigated algorithms.

The paper is organized as follows. Section II describes the system identification model and reviews the conventional LMMN algorithm. In Section III, the proposed modified LMMN algorithm is derived which is robust under impulsive environments. To promote system sparsity, ZA-SLMMN and RZA-SLMMN sparse algorithms are proposed in Section IV. Simulation results are shown in Section V. Section VI presents the conclusion of the paper.

II. Description Of System Model And LMMN Algorithm

Let us consider the system identification model as shown in Fig. 1.

The output of the unknown system is described as

$$d(n) = \boldsymbol{x}^T(n)\boldsymbol{W}_{opt} + v(n), \tag{1}$$

where, $\boldsymbol{x}(n) = [x(n), x(n-1), ..., x(n-L+1)]^T$ denotes an $Lx1$ input signal vector and $\boldsymbol{W}_{opt} = [w_0, w_1, ..., w_{L-1}]^T$ is an unknown system weight vector. The system noise $v(n)$ consists of white Gaussian noise and impulsive noise. The error signal is defined as

$$e(n) = d(n) - y(n) = d(n) - \boldsymbol{x}^T(n)\boldsymbol{W}(n-1), \tag{2}$$

where, $y(n)$ is the output of the adaptive filter $\boldsymbol{W}(n)$.

The cost function of the LMMN algorithm is given by

$$J_{LMMN}(n) = \frac{\lambda}{2}E\{e^2(n)\} + \frac{1-\lambda}{4}E\{e^4(n)\} \tag{3}$$

which is a combination of LMS and LMF algorithm cost functions and λ is the mixing parameter, $0 \le \lambda \le 1$.

The LMMN weight update equation is

$$\begin{aligned}\boldsymbol{W}(n+1) &= \boldsymbol{W}(n) - \mu\hat{\nabla}_{\boldsymbol{W}(n)}J_{LMMN}(n) \\ &= \boldsymbol{W}(n) + \mu e(n)\{\lambda + (1-\lambda)e^2(n)\}\boldsymbol{x}(n),\end{aligned} \tag{4}$$

where μ is the step-size of LMMN algorithm.

III. Modified LMMN Algorithm Based On Sigmoid Function

The sigmoid function is defined as in [25], [26]

$$S(n) = sgm[\alpha J_{LMMN}(n)] = \frac{1}{1+e^{-\alpha J_{LMMN}(n)}}, \tag{5}$$

where α is the steepness parameter of the sigmoid function.

The basic idea of using sigmoid is to exploit the saturation property of the nonlinearity of sigmoid function. The same framework was used in [19] and several robust adaptive filtering algorithms for impulsive noise scenarios were proposed.

The modified cost function of the LMMN algorithm based on (5) is given by

$$J_{SLMMN}(n) = \frac{1}{\alpha}S(n) = \frac{1}{\alpha}\frac{1}{1+e^{-\alpha J_{LMMN}(n)}}. \tag{6}$$

On differentiating the sigmoid LMMN (SLMMN) cost function (6) with respect to $\boldsymbol{W}(n)$ yields

$$\begin{aligned}\hat{\nabla}_{\boldsymbol{W}(n)}J_{SLMMN}(n) &= \frac{\partial J_{SLMMN}(n)}{\partial \boldsymbol{W}(n)} \\ &= \frac{1}{\alpha}\frac{\partial S(n)}{\partial J_{LMMN}(n)}\frac{\partial J_{LMMN}(n)}{\partial \boldsymbol{W}(n)} \\ &= S(n)[1-S(n)]\hat{\nabla}_{\boldsymbol{W}(n)}J_{LMMN}(n)\end{aligned} \tag{7}$$

The weight update equation of the proposed SLMMN algorithm is given by

$$\boldsymbol{W}(n+1) = \boldsymbol{W}(n) - \mu\hat{\nabla}_{\boldsymbol{W}(n)}J_{SLMMN}(n). \tag{8}$$

Substituting (7) into (8), we obtain

$$\begin{aligned}\boldsymbol{W}(n+1) &= \boldsymbol{W}(n) - \mu S(n)[1-S(n)]\hat{\nabla}_{\boldsymbol{W}(n)}J_{LMMN}(n) \\ &= \boldsymbol{W}(n) + \mu S(n)[1-S(n)]e(n)\{\lambda + (1-\lambda)e^2(n)\}\boldsymbol{x}(n),\end{aligned} \tag{9}$$

$$\begin{aligned}\text{where, } S(n) &= sgm\left[\alpha\left\{\frac{\lambda}{2}\left(e^2(n)\right) + \frac{1-\lambda}{4}\left(e^4(n)\right)\right\}\right] \\ &= \frac{1}{1+e^{-\alpha\left\{\frac{\lambda}{2}\left(e^2(n)\right)+\frac{1-\lambda}{4}\left(e^4(n)\right)\right\}}}.\end{aligned} \tag{10}$$

IV. Proposed Sparse SLMMN Algorithms

To exploit the system sparsity, two sparse algorithms are proposed by introducing the ℓ1-norm and log-sum penalties into the SLMMN namely, Zero Attracting SLMMN (ZA-SLMMN) and Reweighted Zero Attracting SLMMN (RZA-SLMMN) algorithms respectively.

A. Zero Attracting SLMMN (ZA-SLMMN) Algorithm

The cost function of ZA-SLMMN algorithm is the following

$$J_{ZA-SLMMN}(n) = J_{SLMMN}(n) + \gamma_{ZA}\|\boldsymbol{W}(n)\|_1 \tag{11}$$

where γ_{ZA} is the regularization parameter balancing the estimation error and $\|\boldsymbol{W}(n)\|_1$.

The ZA-SLMMN algorithm update is defined as

$$\boldsymbol{W}(n+1) = \boldsymbol{W}(n) - \mu\hat{\nabla}_{\boldsymbol{W}(n)}J_{ZA-SLMMN}(n) \tag{12}$$

where,

$$\begin{aligned}\hat{\nabla}_{\boldsymbol{W}(n)}J_{ZA-SLMMN}(n) &= \frac{\partial J_{ZA-SLMMN}(n)}{\partial \boldsymbol{W}(n)} \\ &= \frac{\partial J_{SLMMN}(n)}{\partial \boldsymbol{W}(n)} + \gamma_{ZA}sgn(\boldsymbol{W}(n))\end{aligned} \tag{13}$$

Using (7) in (13) and substituting into (12), we obtain the weight updation of the proposed ZA-SLMMN algorithm.

$$W(n+1)=W(n)+\mu S(n)[1-S(n)]e(n)\left|\lambda+(1-\lambda)e^2(n)\right|x(n) - \rho_{ZA} sgn(W(n)), \quad (14)$$

where, $sgn(.)$ is the sigmum function, $\rho_{ZA}=\mu\gamma_{ZA}$, and

$$S(n)=sgm\left[\alpha\left\{\frac{\lambda}{2}\left(e^2(n)\right)+\frac{1-\lambda}{4}\left(e^4(n)\right)+\gamma_{ZA}\|W(n)\|_1\right\}\right] \quad (15)$$

B. Reweighted Zero Attracting SLMMN (RZA-SLMMN) Algorithm

The cost function of RZA-SLMMN algorithm is obtained by introducing the log-sum penalty [27] into the SLMMN cost function as follows:

$$J_{RZA\text{-}SLMMN}(n)=J_{SLMMN}(n)+\gamma_{RZA}\sum_{i=0}^{L-1}\log\left(1+\varepsilon_{RZA}|w_i(n)|\right) \quad (16)$$

where γ_{RZA} is the regularization parameter.

The weight update equation of RZA-SLMMN algorithm is derived as

$$W(n+1)=W(n)-\mu\tilde{\nabla}_{W(n)}J_{RZA\text{-}SLMMN}(n). \quad (17)$$

On differentiating the second term in (17) with respect to $W(n)$, yields the following equation that corresponds to the weight updation of the proposed RZA-SLMMN algorithm.

$$W(n+1)=W(n)+\mu S(n)[1-S(n)]e(n)\left|\lambda+(1-\lambda)e^2(n)\right|x(n) - \rho_{RZA}\frac{sgn(W(n))}{1+\varepsilon_{RZA}|W(n)|}, \quad (18)$$

where, $\rho_{RZA}=\mu\gamma_{RZA}\varepsilon_{RZA}$.

$$S(n)=sgm\left[\alpha\left\{\frac{\lambda}{2}\left(e^2(n)\right)+\frac{1-\lambda}{4}\left(e^4(n)\right)+\sum_{i=0}^{L-1}\log\left(1+\varepsilon_{RZA}|w_i(n)|\right)\right\}\right] \quad (19)$$

V. SIMULATION RESULTS

The performance of the proposed SLMMN algorithms is evaluated in the system identification scenario. The Matlab codes of the proposed algorithms are available at http://falbu.50webs.com/tsp2019/codes.rar.

The unknown system and the filter length are set to L = 16 and the system sparsity of K is chosen from {1, 4, 8}. The correlated (colored) input signal is generated by filtering a unit variance $\sigma_x^2=1$ (0 dB) Gaussian white noise through an AR(1) first-order autoregressive system having a pole at 0.8. The system noise $v(n)$ contains white Gaussian noise with SNR = 20dB and Bernoulli-Gaussian (B-G) distributed impulsive noise generated as $q(n)=b(n)v_i(n)$. The binary process, $b(n)$ is described by the probability $p(b(n)=1)=P$, $p(b(n)=0)=1-P$, where P is the probability of occurrence of the impulsive noise [16]. $v_i(n)$ is assumed to be a zero-mean white Gaussian noise with variance $\sigma_{v_i}^2$. The normalized mean-square deviation (NMSD) is used to estimate the performance of the proposed algorithms and is defined as follows:

$$NMSD(n)=10\log_{10}\frac{\left\|W_{opt}-W(n)\right\|_2^2}{\left\|W_{opt}\right\|_2^2}(dB) \quad (20)$$

The results for 100 trials are averaged in all simulations. In order to show the effectiveness of the proposed SLMMN algorithms, a comparison with the LMP algorithms is performed. The simulation parameters setting for the proposed algorithms are as follows:

$$\mu=0.04,\ P=0.01,\ \lambda=0.5,\ \sigma_{v_i}^2=10^4/12,\ \alpha=0.6,$$
$$\rho_{ZA}=5\text{x}10^{-5},\ \rho_{RZA}=1\text{x}10^{-4},\ \text{and}\ \varepsilon_{RZA}=20.$$

From Figs. 2, 3 and 4, it can be seen that the SLMMN, ZA-SLMMN and RZA-SLMMN yield better steady state performances than the sparse LMP algorithms (ZA-LMP and RZA-LMP), while the LMMN algorithm does not converge in the presence of impulsive noise. Hence, the proposed algorithms are robust against impulsive noise and are capable of handling the system with different sparsity levels, K={1, 4, 8}. The RZA-SLMMN algorithm exhibits superior performance and achieves the lowest steady-state error in all the cases.

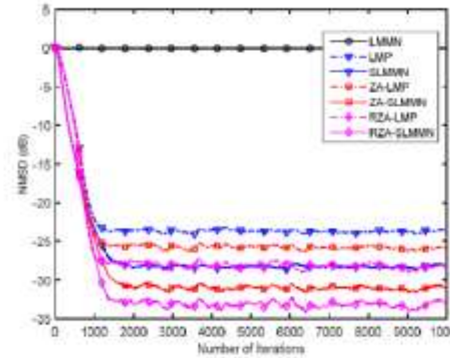

Fig. 2. NMSD comparison of the proposed SLMMN algorithms for the system with sparsity $K=1$ and in the presence of impulsive noise

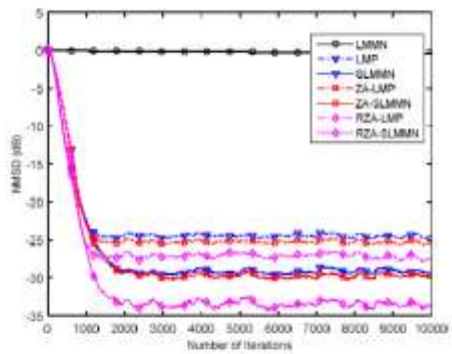

Fig. 3. NMSD comparison of the proposed SLMMN algorithms for the system with sparsity K =4 and in the presence of impulsive noise

It can be noticed from Fig. 5 that increasing the step-size value μ leads to an increased convergence rate of the proposed SLMMN algorithm, but also results in high steady-state error.

As can be seen from Fig. 6, the greater the steepness parameter α of the sigmoid function, the lower is the steady-state error misadjustment and slower the convergence of the SLMMN. Depending on the particular practical application, the proper choice of the parameters of the SLMMN algorithm can highly reduce the number of needed iterations if a certain NMSD performance is needed.

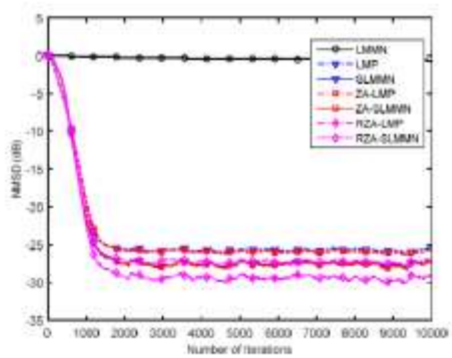

Fig. 4. NMSD comparison of the proposed SLMMN algorithms for the system with sparsity K =8 and in the presence of impulsive noise

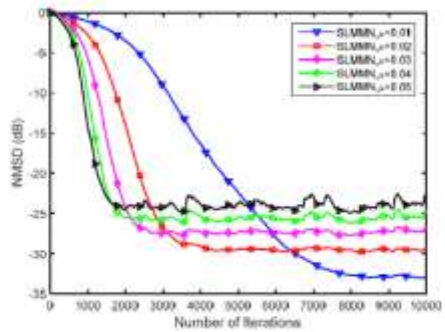

Fig. 5. NMSD of the proposed SLMMN algorithm with different step-size parameter μ

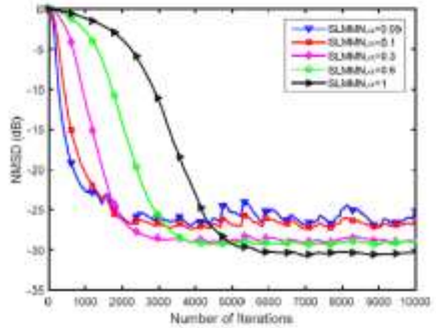

Fig. 6. NMSD of the proposed SLMMN algorithm with different α

VI. Conclusion

It is known that the existing LMMN algorithm fails to converge in the presence of non-Gaussian impulsive interferences. Several algorithms using the sigmoid function are proposed: the SLMMN, the ZA-SLMMN and the RZA-SLMMN algorithms. It is shown that the proposed algorithms are capable of combating impulsive noise and estimating effectively the system with different levels of sparsity and achieves a lower steady-state misadjustment when compared with competing algorithms.

Acknowledgments

The authors thank the anonymous reviewers for their useful comments that helped us to improve the paper.

References

[1] S. Haykin, Adaptive Filter Theory, 4th ed. Upper Saddle River, NJ: Prentice-Hall, 2002.

[2] A.H. Sayed, Fundamentals of Adaptive Filtering, New York: Wiley-Interscience, 2003.

[3] J. Benesty and Y. Huang, Adaptive Signal Processing-Applications to Real World Problems. Berlin, Germany: Springer-Verlag, 2003.

[4] B. Widrow and S. D. Stearns, Adaptive Signal Processing, New Jersey: Prentice Hall, 1985.

[5] E. Walach and B. Widrow, "The least mean fourth (LMF) adaptive algorithm and its family," IEEE Trans. Inf. Theory, vol. 30, no. 2, pp. 275-283, Mar. 1984.

[6] J.A. Chambers, O. Tanrikulu, and S.A.G. Constantinides, "Least mean mixed-norm adaptive filtering," Electronics Letters, vol.30, no.19, pp. 1574-1575, 1994.

[7] I. Pitas and A.N. Venetsanopoulos, "Adaptive filters based on order statistics," IEEE Trans. Signal Process., vol. 39, no. 2, pp. 518-522, Feb. 1991.

[8] P. Salembier, "Adaptive rank order based filters," Signal Process., vol. 27, no. 1, pp. 1-25, Apr. 1992.

[9] M. Shao and C.L. Nikias, "Signal processing with fractional lower order moments: Stable processes and their applications," Proc. IEEE, vol. 81, no. 7, pp. 986–1010, Jul. 1993.

[10] S.V. Vaseghi, Advanced Digital Signal Processing and Noise Reduction, 2nd ed. Chichester: John Wiley & Sons Ltd., 2000.

[11] O.Arikan, M.Belge, A.E. Cetin, and E.Erzin, "Adaptive filtering approaches for non-Gaussian stable processes," in Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP), Detroit, MI, pp. 1400-1404, 1995.

[12] S. Koike, "Variable step size normalized sign algorithm for fast convergent adaptive filters with robustness against impulsive noise," NEC Res. Dev., vol. 41, no. 3, pp. 278–288, 2000.

[13] T. Shao, Y.R. Zheng, and J. Benesty, "A variable step-size normalized sign algorithm for acoustic echo cancelation," in IEEE International Conference on Acoustics, Speech, and Signal Processing, pp. 333–336 2010.

[14] S.C. Pei and C.C. Tseng, "Least mean p-power error criterion for adaptive FIR filter," IEEE J. Sel. Areas Commun., vol. 12, no. 9, pp. 1540–1547, 1994.

[15] B. Weng and K.E. Barner, "Nonlinear system identification in impulsive environments," IEEE Trans. Signal Process., vol. 53, no. 7, pp. 2588–2594, 2005.

[16] J. Chambers, A. Avlonitis, "A robust mixed-norm adaptive filter algorithm," IEEE Signal Process. Lett., vol. 4, no. 2, pp. 46–48, 1997.

[17] R. Pogula, T. Kishore Kumar, and F. Albu, "Complex domain adaptive system identification using sparse affine projection normalized correlation algorithms under impulsive noises," in Proc. of COMM 2018, Bucharest, Romania, pp. 61-66, 2018.

[18] R. Pogula, T. Kishore Kumar, and F. Albu, "Novel sparse algorithms based on Lyapunov stability for adaptive system identification," Radioengineering, vol. 27, no. 1, pp. 270-280, 2018.

[19] F. Huang, J. Zhang, and S. Zhang, "A family of robust adaptive filtering algorithms based on sigmoid cost," Signal Processing, vol. 149, pp. 179-192, 2018.

[20] Y. Chen, J. Tian, and Y. Liu, "Variable step size LMS algorithm based on modified sigmoid function," 2014 International Conference on Audio, Language and Image Processing, Shanghai, pp. 627-630, 2014.

[21] S.C. Douglas and T.H.Y. Meng, "An adaptive edge detection method using a modified sigmoid-LMS algorithm," 23rd Asilomar Conference on Signals, Systems and Computers, Pacific Grove, USA, pp. 252-256, 1989.

[22] W. Bajwa, J. Haupt, G. Raz, and R. Nowak, "Compressed channel sensing," in Prof. IEEE CISS, pp. 5-10, 2008.

[23] Y. Li, Y. Wang, and F. Albu, "Sparse channel estimation based on a reweighted least-mean mixed-norm adaptive filter algorithm," in Proc. of EUSIPCO 2016, Budapest, Hungary, pp. 2380-2384, 2016.

[24] W. Ma, B. Chen, H. Qu, and J. Zhao, "Sparse least mean p-power algorithms for channel estimation in the presence of impulsive noise," Signal, Image and Video Processing, vol. 10, no. 3, pp. 503–510, 2016.

[25] J.C. Principe, N.R. Euliano, and W.C. Lefebvre, Neural and Adaptive Systems: Fundamentals through Simulations. New York: John Wiley & Sons, 2000.

[26] S. Haykin, Neural Networks and Learning Machines, 3rd ed. Upper Saddle River, NJ: Pearson Education, 2009.

[27] E.J. Candès, M.B. Wakin, and S.P. Boyd, "Enhancing sparsity by reweighted ℓ1 minimization," J. Fourier Anal. Appl. vol. 14, no. 5, pp. 877-905, 2008.