Analysis of the LNS Implementation of the Fast Affine Projection algorithms

ESPRIT HSLA PROJECT

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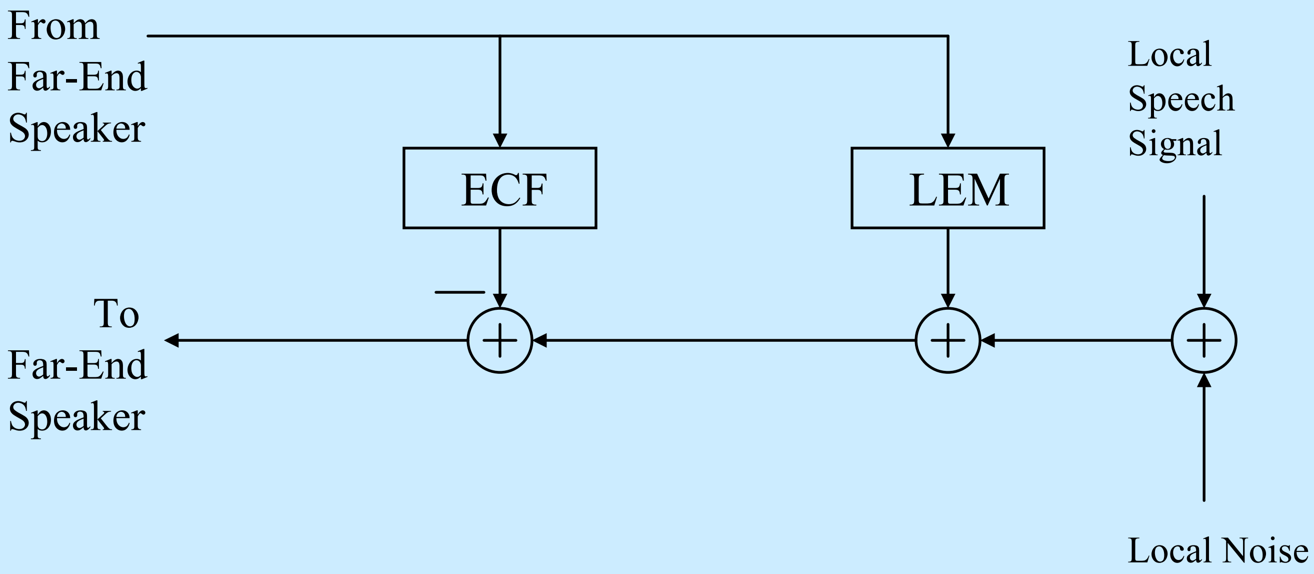
Acoustic echo cancellation

- Loudspeaker-enclosure-microphone (LEM) with an echo-cancellation filter (ECF)

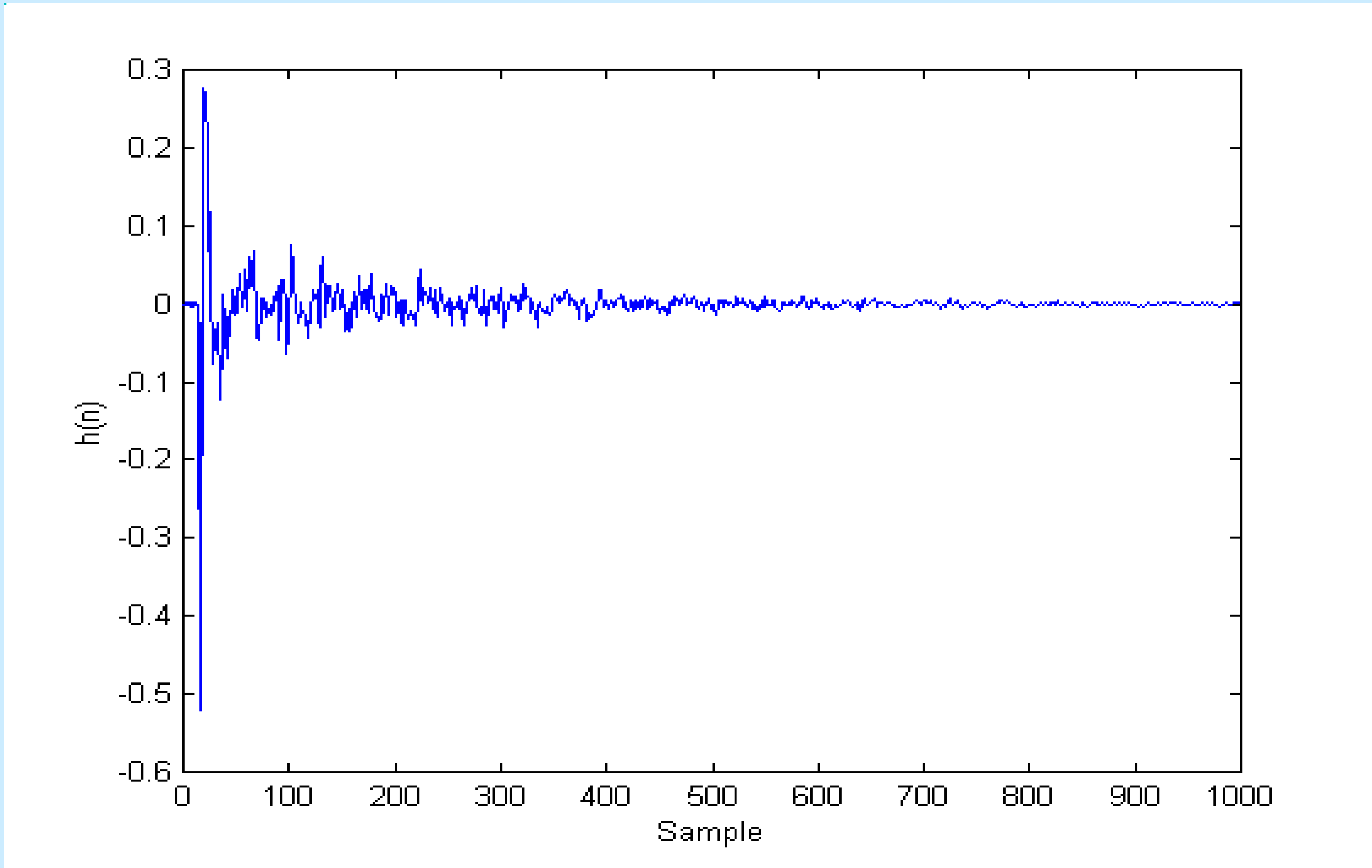


Acoustic echo cancellation

- The echo path is very long (~125 ms)
- The echo path may rapidly change at any time
- The impulse response varies with ambient temperature, pressure, humidity, movement of objects

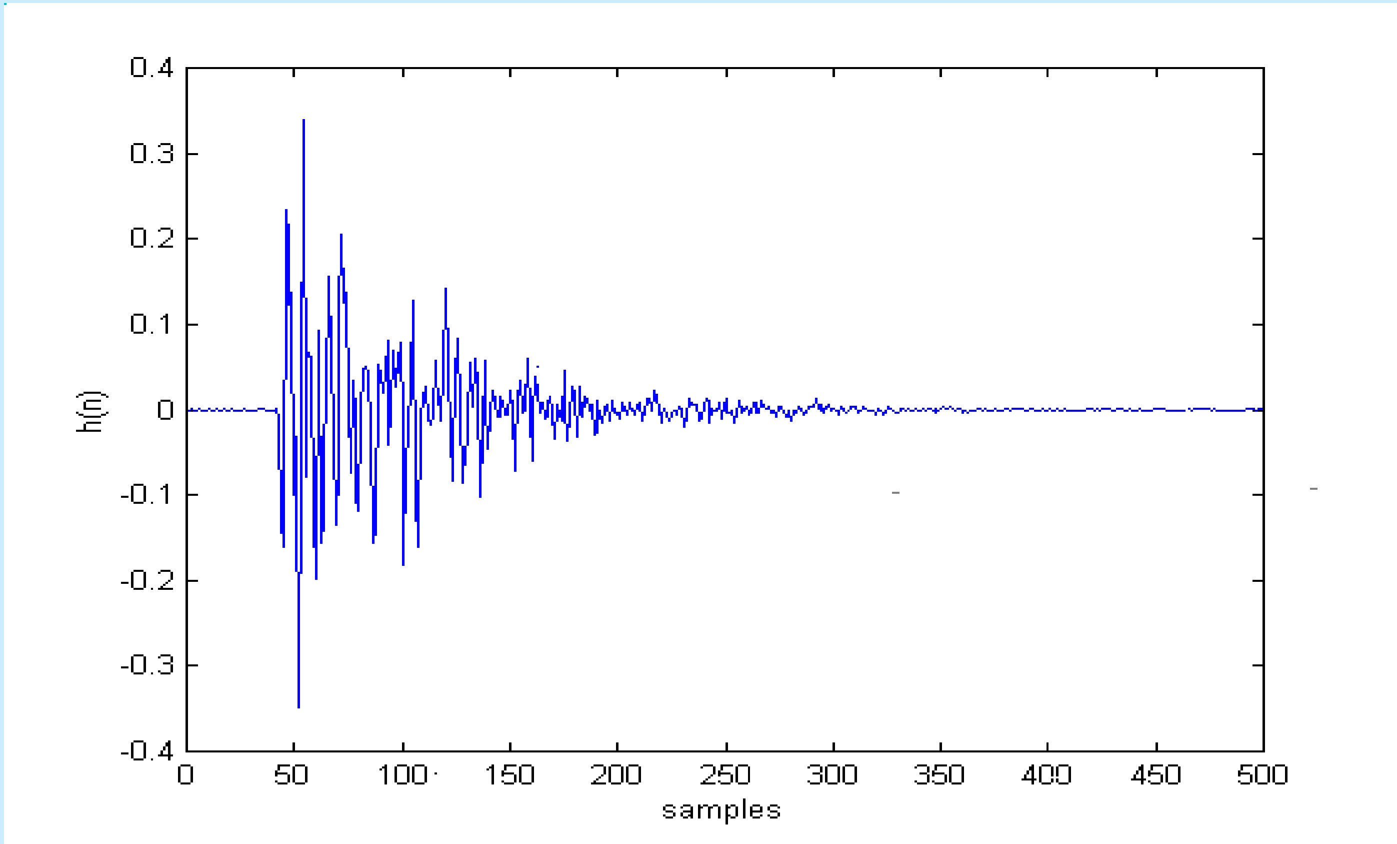
Acoustic echo cancellation

- The room impulse response



Acoustic echo cancellation

- The car impulse response



Logarithmic number system

IEEE single precision:

S	Exponent	Mantissa
31	30 … 23	22 … 0

32b LNS:

S	Exp integer	Exp fraction part
31	30 … 23	22 … 0

Elementary LNS operations:

x + y	ADD	Lz=Lx+log(1+2^(Ly-Lx)), Sz depends on sizes of x, and y
x - y	SUB	Lz=Lx+log(1- 2^(Ly-Lx)), Sz depends on sizes of x and y
x * y	MUL	Lz=Lx+Ly, Sz=Sx OR Sy
x / y	DIV	Lz =Lx-Ly, Sz=Sx OR Sy
x^0.5	SQRT	Lx >> 1, Sz=Sx

FAP Algorithms

Affine Projection Algorithm (APA) is a generalisation of the NLMS algorithm

1) $\underline{e}_n = \underline{s}_n - \mathbf{X}_n^t \underline{h}_{n-1}$

2) $\underline{\varepsilon}_n = \left[\mathbf{X}_n^t \mathbf{X}_n + \delta \mathbf{I}\right]^{-1} \underline{e}_n$

3) $\underline{h}_n = \underline{h}_{n-1} + \mu_A \mathbf{X}_\mathbf{n} \underline{\varepsilon}_n$

The complexity of APA is $2LN + O(N^2)$ where L is the length of the adaptive filter, N is the size of the projection .

FAP Algorithms

0) Initialization: $\underline{a}_0 = \left[1, \underline{0}^t\right]^t, \underline{b}_0 = \left[\underline{0}^t, 1\right]^t, \; E_{a,n} = E_{b,n} = \delta$	
1) Use sliding windowed FTF algorithm to update $E_{a,n}, E_{b,n}, \underline{a}_n$, and $\underline{b}_n$	10N
2) $\underline{\tilde{r}}_{xx,n} = \underline{\tilde{r}}_{xx,n-1} + x_n \underline{\tilde{\alpha}}_n - x_{n-L} \underline{\tilde{\alpha}}_{n-L}$	2N
3) $\hat{e}_n = s_n - \underline{x}_n^t \underline{\hat{h}}_{n-1}$	L
4) $e_n = \hat{e}_n - \mu \underline{\tilde{r}}_{xx,n}^t \overline{\underline{E}}_{n-1}$	N
5) $\underline{e} = \begin{bmatrix} e_n \\ (1-\mu)\overline{\underline{e}}_{n-1} \end{bmatrix}$	N
6) $\underline{\varepsilon} = \begin{bmatrix} 0 \\ \underline{\tilde{\varepsilon}}_n \end{bmatrix} + \frac{1}{E_{a,n}} \underline{a}_n \underline{a}_n^t \underline{e}$	2N
7) $\begin{bmatrix} \overline{\underline{\varepsilon}}_n \\ 0 \end{bmatrix} = \underline{\varepsilon}_n - \frac{1}{E_{b,n}} \underline{b}_n \underline{b}_n^t e_n$	2N
8) $\underline{E}_n = \begin{bmatrix} 0 \\ \overline{\underline{E}}_{n-1} \end{bmatrix} + \underline{\varepsilon}_n$	N
9) $\underline{\hat{h}}_n = \underline{\hat{h}}_{n-1} + \mu \underline{x}_{n-(N-1)} E_{N-1,n}$	L
10) $\underline{\hat{\varepsilon}}_{n+1} = (1-\mu)\overline{\underline{\varepsilon}}_n$	N

Total: $2L + 20N$

CGFAP Algorithm

Initialisation **(Conjugate Gradient FAP algorithm)**

$$0.\ \underline{V}(-1) = \underline{0},\ \underline{\eta}(-1) = 0,\ \underline{s}(-1) = 0,\ \mathbf{R}(-1) = \delta\mathbf{I},\ \alpha = 1,\ \underline{P}(-1) = \underline{b}/\delta$$

Processing in sampling interval *n*

1) $\mathbf{R}(n) = \mathbf{R}(n-1) + \underline{\xi}(n)\underline{\xi}^T(n) - \underline{\xi}(n-L)\underline{\xi}^T(n-L)$

2) $\underline{g}(n) = \mathbf{R}(n)\underline{P}(N-1) - \underline{b}$

3) $\gamma(n) = \dfrac{\underline{g}^T(n)\mathbf{R}(n-1)\underline{s}(n-1)}{\underline{s}^T(n-1)\mathbf{R}(n-1)\underline{s}(n-1)}$

4) $\underline{s}(n) = \gamma(n)\underline{s}(n-1) - \underline{g}(n)$

5) $\underline{P}(n) = \underline{P}(n-1) - \dfrac{\underline{g}^T(n)\underline{s}(n)}{\underline{s}^T(n)\mathbf{R}(n)\underline{s}(n)}\underline{s}(n)$

6) $\underline{V}(n) = \underline{V}(n-1) + \alpha\eta_{N-1}(N-1)\underline{X}(n-N)$

7) $y(n) = \underline{V}^T(n)\underline{X}(n) + \alpha\underline{\overline{\eta}}^T(n-1)\underline{\tilde{R}}(n)$

8) $e(n) = d(n) - y(n)$

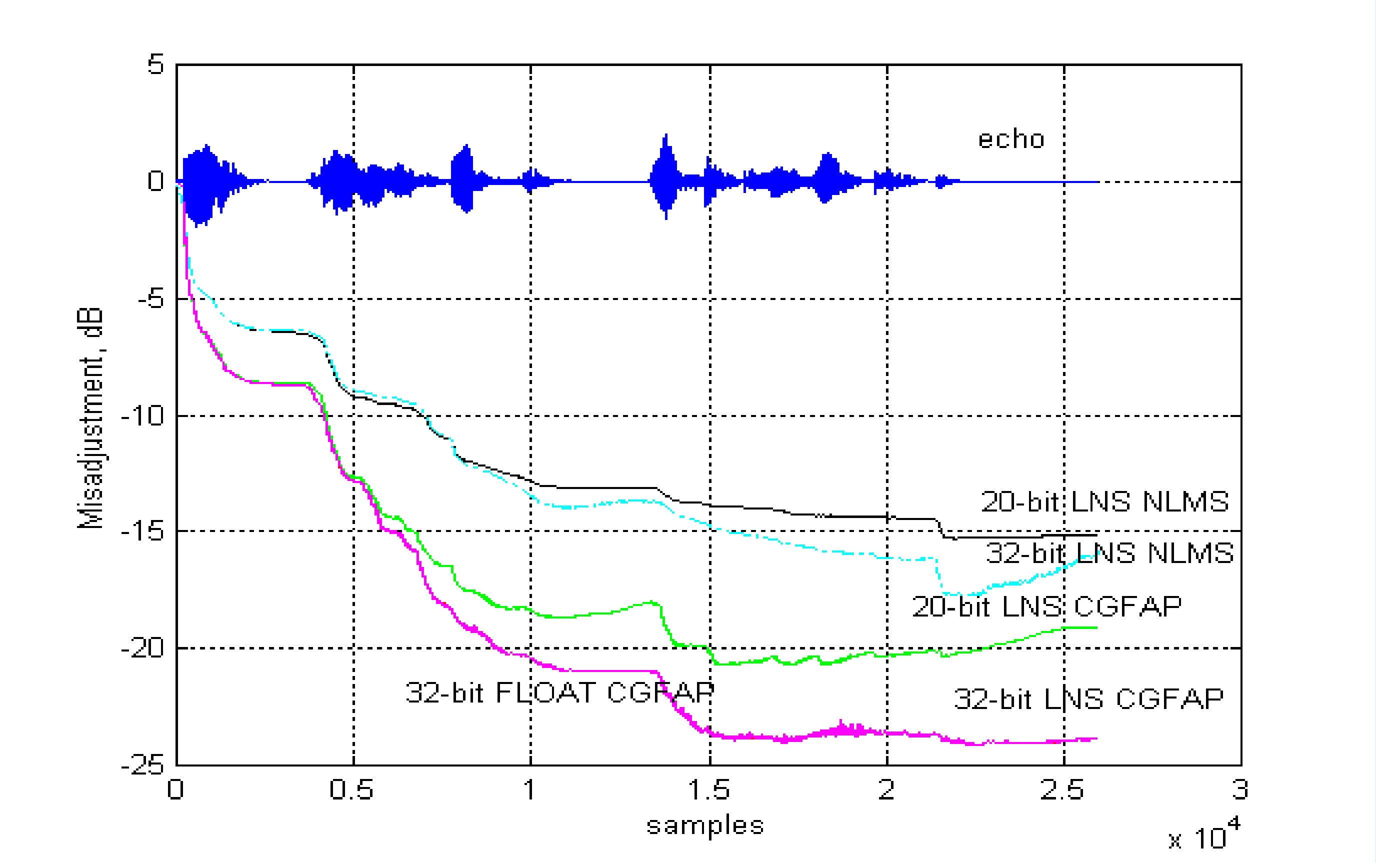
9) $\underline{\varepsilon} = e(n)\underline{P}(n)$

10) $\underline{\eta}(n) = \begin{bmatrix} 0 \\ \underline{\overline{\eta}}(n-1) \end{bmatrix} + \underline{\varepsilon}(n)$

Total : $2L + 2N^2 + 9N + 1$ (1 division)

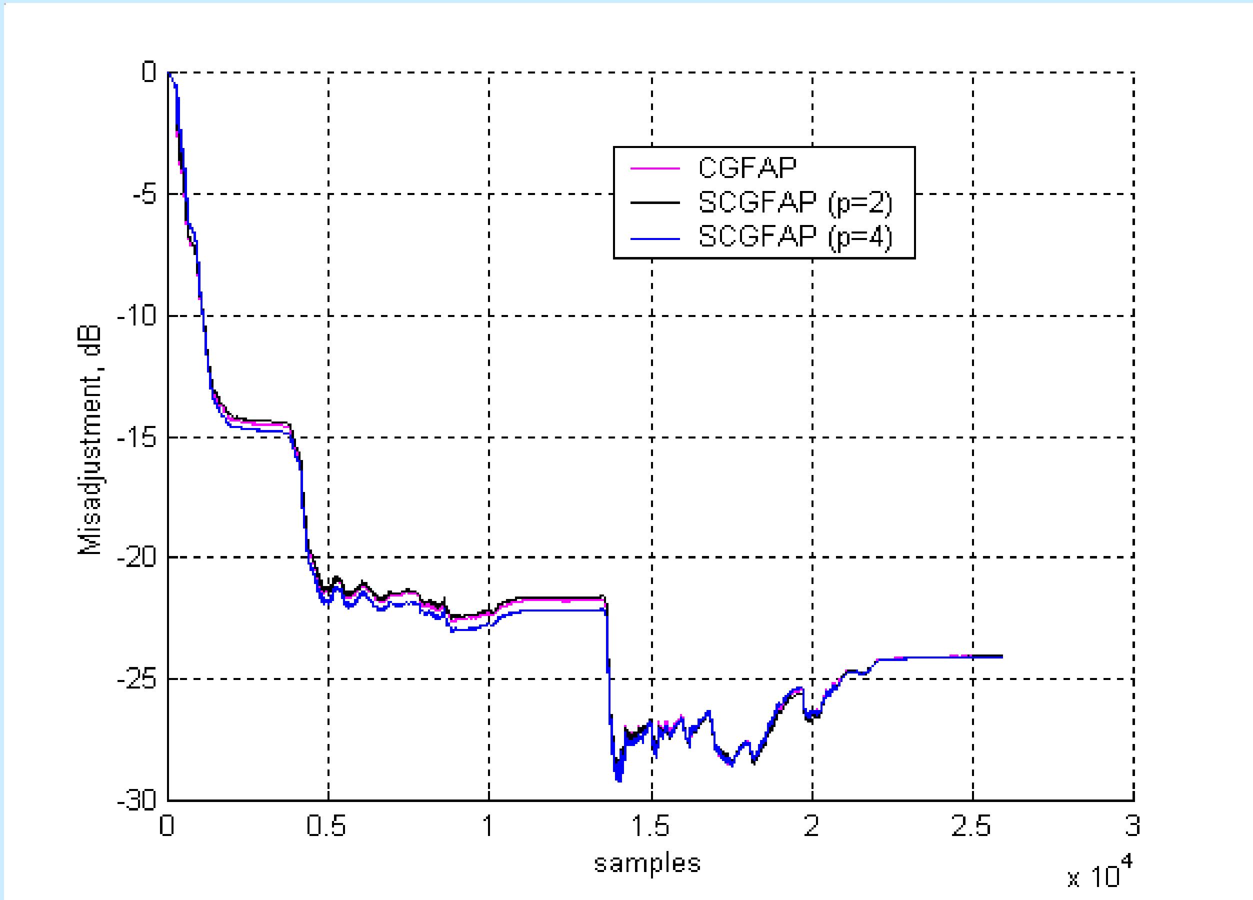
Simulations

The learning curves for 32-bit FLOAT, 32-bit and 20-bit LNS implementations of CGFAP algorithm (32-bit curves almost co-incidental) and DOUBLE NLMS algorithm (L=1000, N=10)



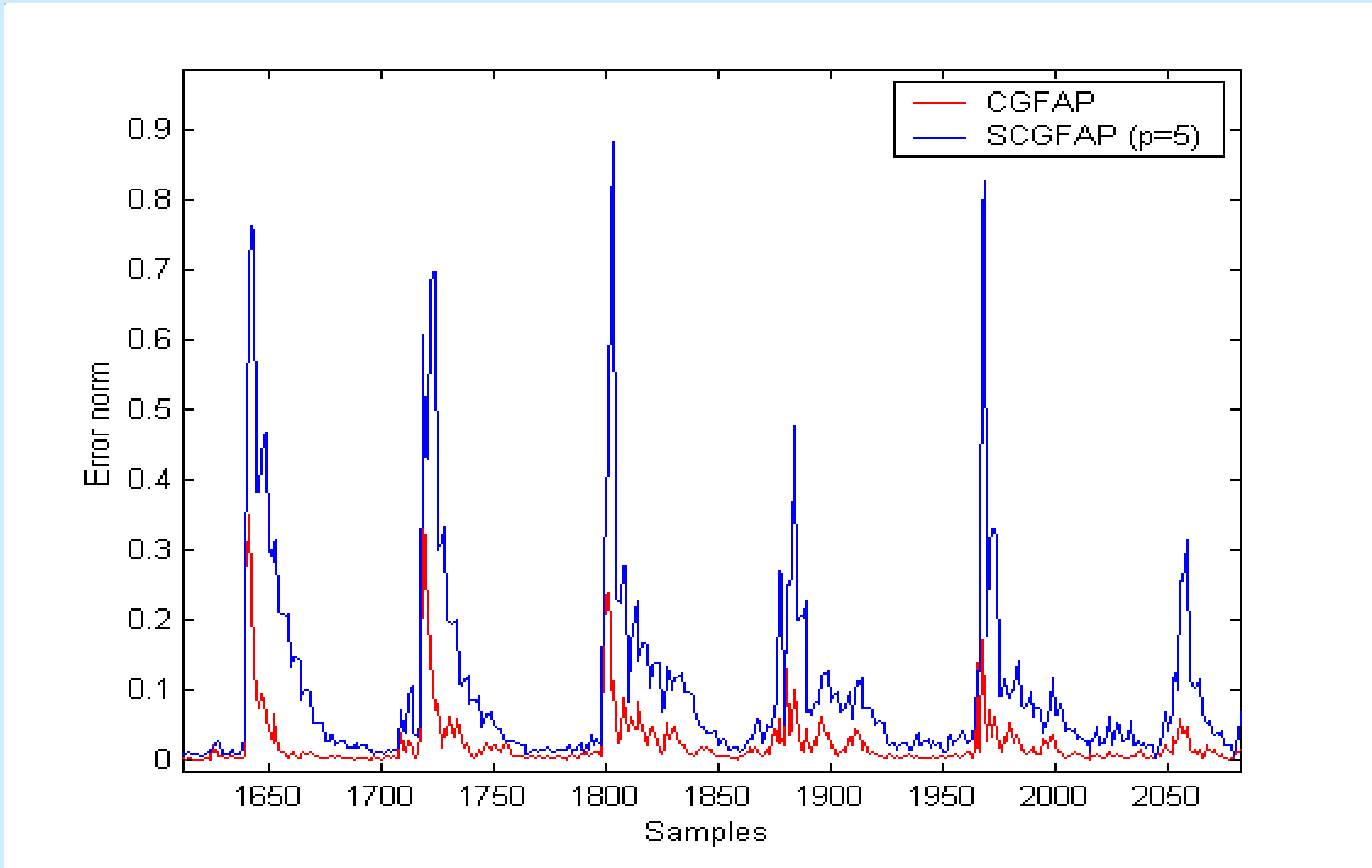
Simulations

Convergence of 20-bit LNS CGFAP implementation for different values of *p* (L=256, N=10)



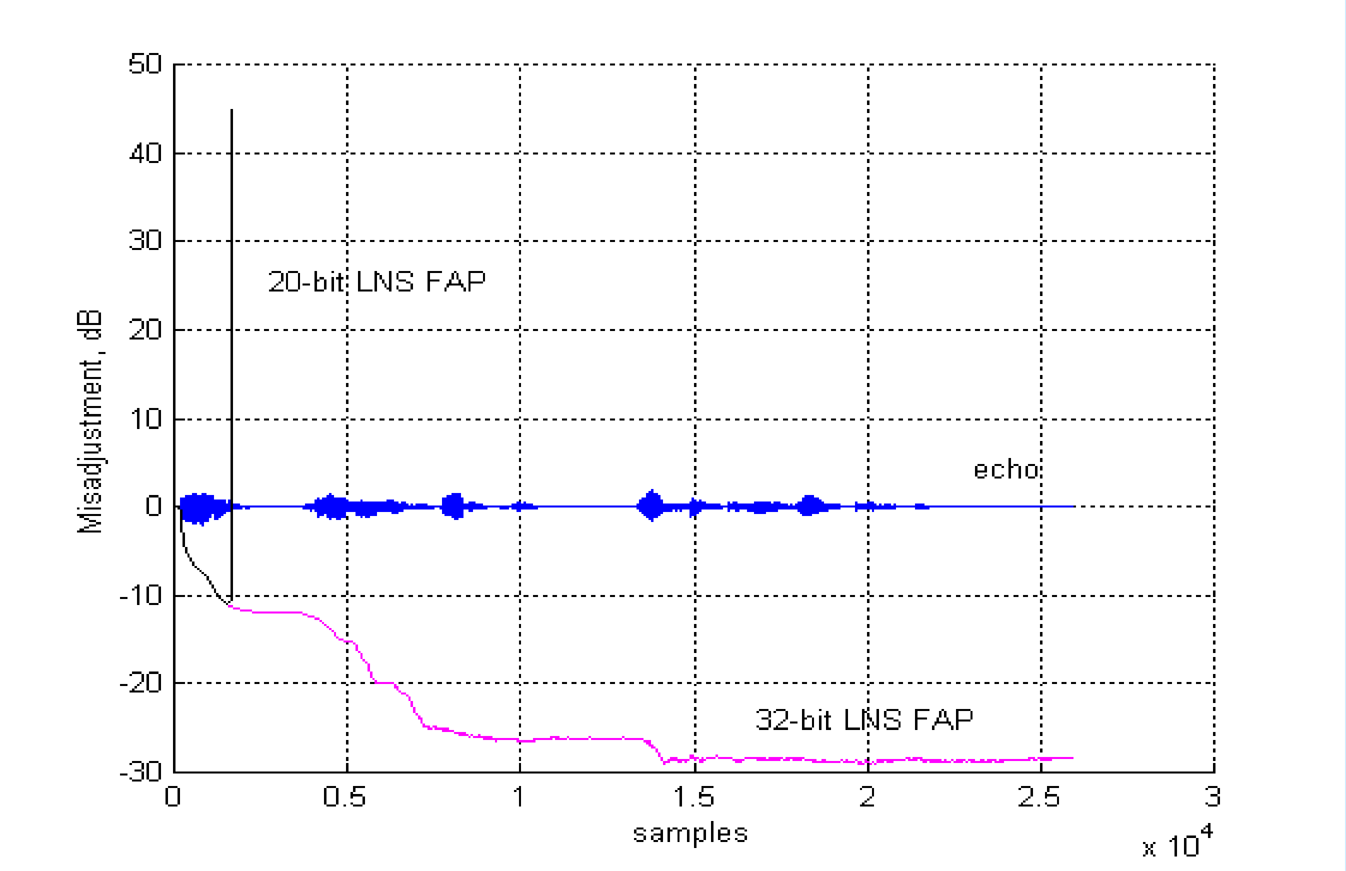
Simulations

The error norm between the exact solution (double precision) and the iterated solution of the linear system for different values of p (p=1 and p=5)



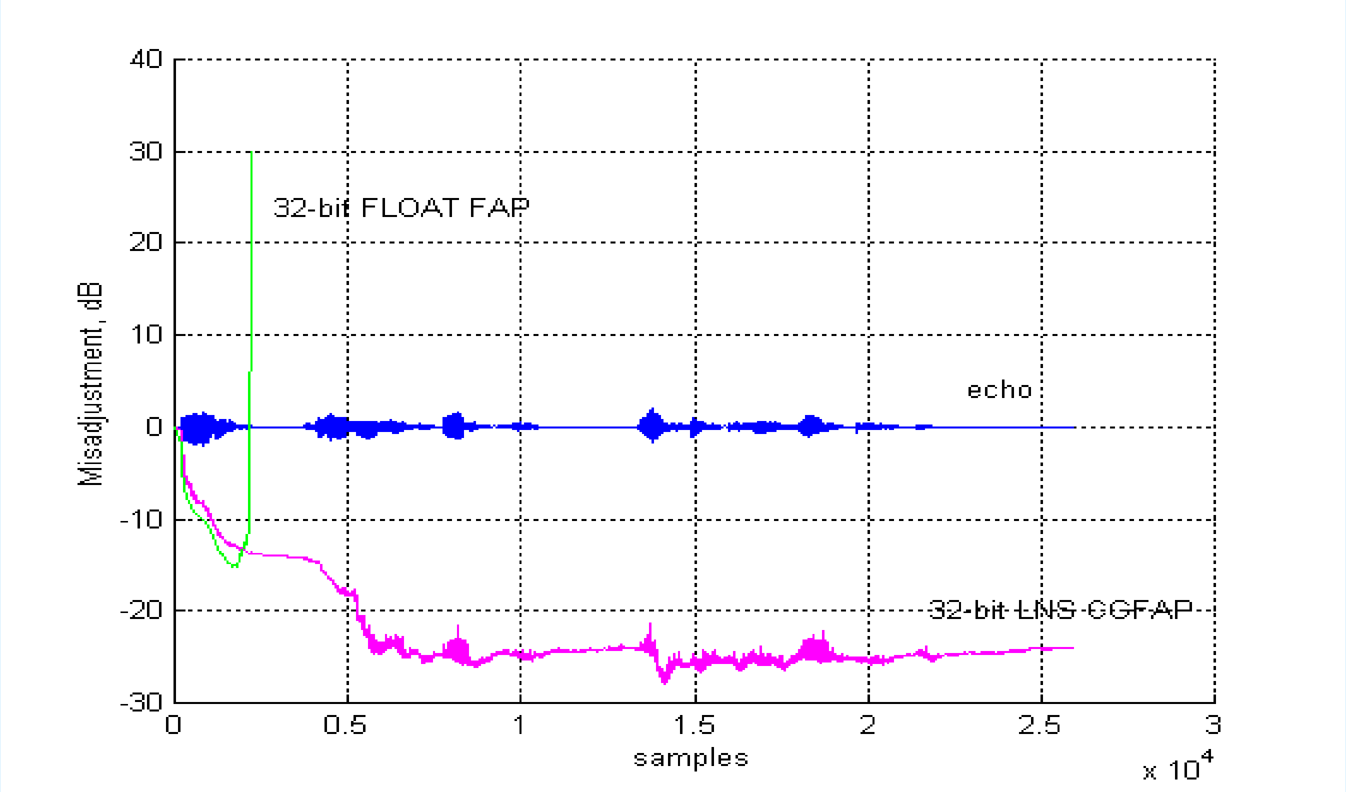
Simulations

Convergence of 32-bit LNS FAP implementation versus 20-bit FLOAT FAP implementation, Float is unstable after about 1600 iterations (L=256, N=10, k=100)



Simulations

- Convergence of 32-bit LNS CGFAP implementation versus 32-bit FLOAT FAP implementation, Float is unstable after about 2200 iterations (L=256, N=10, k=5)



Simulations

We can update $\underline{P}(n)$ less frequently without affecting too much the output error. Therefore, the average number of MACs is

$$2L+2N^2/p+(4+5/p)N-1+2/p$$

If L=1000 and N=10, NLMS needs 2025 MACs (assuming 25 MACs for a division)

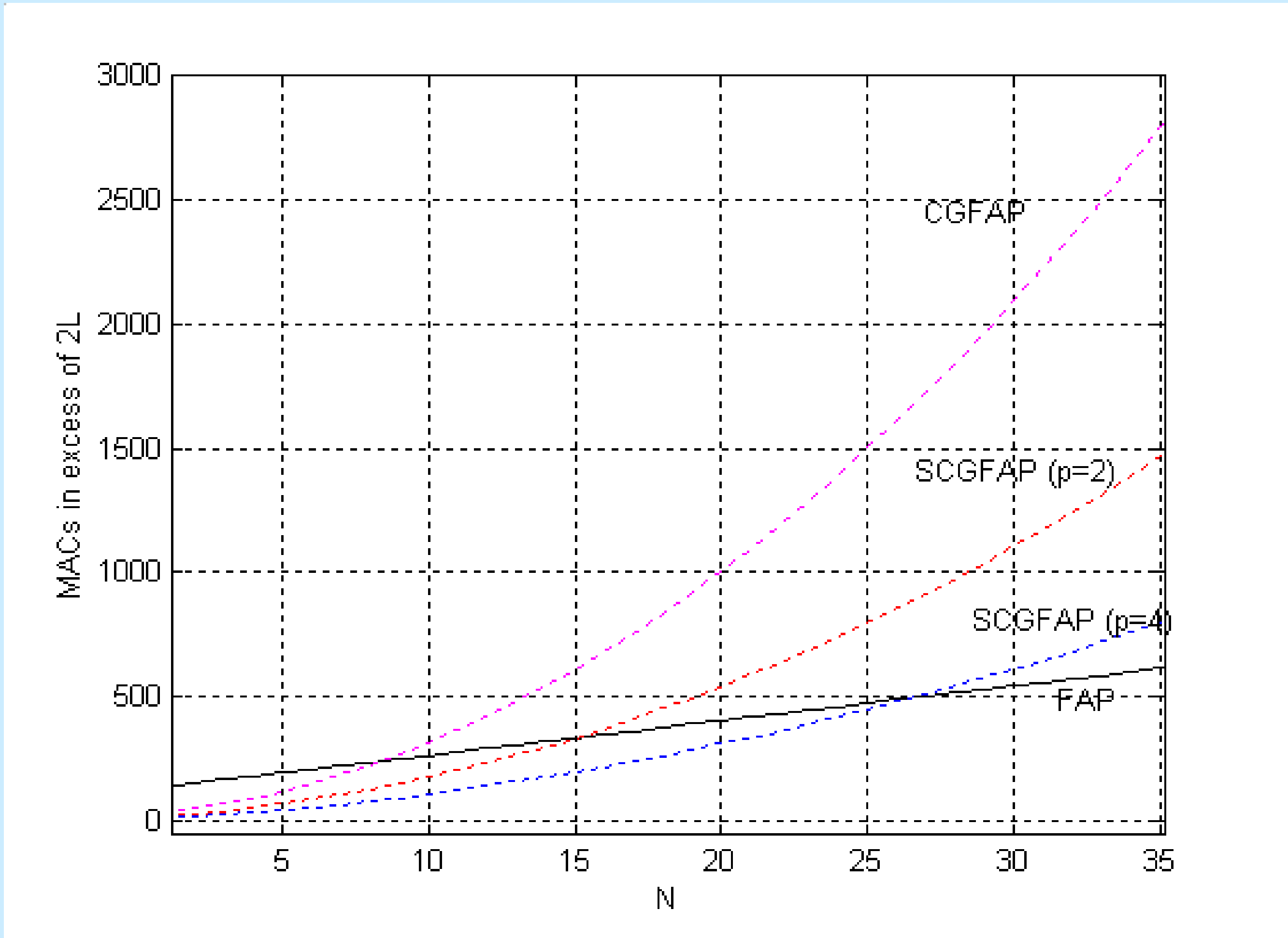
-FAP needs 2265 FAPs ($2L+20N$, 5 divisions)

- CGFAP needs 2316 MACs ($2L+2N^2+9N+1$, 1 division)

- SCGFAP needs 2108 MACs ($2L+2N^2/p+(4+5/p)N-1+2/p$, p=4)

Simulations

Real time requirements of 3 Fast Affine Projection algorithms



Conclusions

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Conclusions

- The SCGFAP Algorithm is a stable FAP algorithm. It is only marginally complex than NLMS, but achieves substantial improvements.
- Its 32-bit and 20-bit LNS are easy to implement. Also, it is suitable to implement with most commercial DSPs because of its reduced memory requirements and low complexity (just 1 division).
- SCGFAP algorithm is a good candidate for different voice applications.

Questions ?

- HSLA project website
  http://napier.ncl.ac.uk/hsla
- UCD's DSP Group website
  http://dsp.ucd.ie